Online Optimal Control of Connected Vehicles for Efficient Traffic Flow at Merging Roads

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Abstract— This paper addresses the problem of coordinating online connected vehicles at merging roads to achieve a smooth traffic flow without stop-and-go driving. We present a framework and a closed-form solution that optimize the acceleration profile of each vehicle in terms of fuel economy while avoiding collision with other vehicles at the merging zone. The proposed solution is validated through simulation and it is shown that coordination of connected vehicles can reduce significantly fuel consumption and travel time at merging roads.

I. INTRODUCTION

The increasing demand for travel has generated significant challenges related to traffic congestion and accidents. Although driver responses to various disturbances can cause congestion [1], intersections and merging roadways are the primary sources of bottlenecks which further contributing to traffic congestion [2]. In the United States, on average 5.5 billion hours are wasted each year due to vehicular congestion, which translates to about $121 billion dollars [3]. In 2012, around 1.7 billion metric tons of CO_2 were released to the environment by vehicles due to congestion [3]. Moreover, traffic congestion can produce driver discomfort, distraction, and frustration, which may encourage more aggressive driving behavior [4] that further slows the process of recovering free traffic flow [5].

Recognition of the necessity for connecting vehicles to their surroundings is gaining momentum. It appears that connected vehicle technologies can improve both transportation network efficiency and safety through the use of control algorithms that harmonically coordinate all existing information. Likewise, communication with traffic structures, nearby buildings, and traffic lights should allow for individual vehicle control systems to account for unpredictable changes in local

infrastructures [6]. A significant research effort has been expended on improving traffic flow at intersections using connected vehicle technologies. In 2008, Dresner and Stone [7] proposed a system to achieve automated vehicle intersection control using a reservation approach. Since then, numerous approaches have been proposed to achieve safe and efficient control of traffic through intersections and merging highways using centralized and decentralized control.

Several research efforts have included a centralized controller or intersection manager that coordinates the vehicles [8]. Zohdy *et al.* [9], Yan *et al.* [10], and Li and Wang [11], focus on the formulation of an optimization problem in which the objective function involves the arrival time at the intersection. The constraints, which are different in each work, are formulated with the goal to avoid collisions. Dynamic programming was applied in [12] to address this problem and a mathematical proof of this approach was presented in [13]. Lee and Park [14] proposed a different approach based on minimizing the overlap in the position of vehicles inside the intersection rather than the arrival time at the intersection. This work was later extended to the case of an urban corridor [15].

Campos *et al.* [16] used a multiobjective optimization framework that includes speed tracking error and acceleration in the objective function to find safe trajectories while satisfying local constraints. Model predictive control (MPC) was used in [17] to solve a multiobjective optimization problem that includes a risk factor function and constraints related to safe velocity and acceleration values. Zohdy and Rakha [18] considered a manager agent and used game theory to address this problem, while Hafner *et al.* [19] took a different path by defining a critical set that can potentially lead to collisions. A more detailed discussion of this approach was presented in [20].

In decentralized control, each vehicle determines its own control policy based on the information received from the other vehicles on the road or some coordinator. Milanes *et al.* [21] used fuzzy logic to design a controller that allows a fully automated vehicle to yield to an incoming vehicle in the conflicting road or cross, if collision risk is not present.

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Milanes *et al.* [22] compared three heuristic intersection control schemes implemented in automated cars. The following year, Makarem *et al.* [23] used MPC to solve the decentralized problem where each vehicle defines its constraints by using some information from other vehicles and solves a linear quadratic control problem accordingly. Jin *et al.* [24] considered platoon formations for decentralized intersection control, where the intersection controller communicates with the platoon leader and the leader with the followers.

In this paper, the objective is to derive a closed-form solution for the problem of coordinating online connected vehicles at merging roads to achieve a smooth traffic flow without stop-and-go driving under the hard constraint of collision avoidance. The contribution of this paper is an analytical solution using the Pontryagin's minimum principle that can be implemented online.

The remainder of this paper is organized as follows: in Section II we discuss the problem for vehicle coordination at merging roads. In Section III we provide the solution to the optimal control problem. Finally, we provide simulation results in Section IV and concluding remarks in Section V.

II. PROBLEM DESCRIPTION

Each vehicle i is described by a second order dynamics model

$$\dot{x}_i = v_i$$
$$\dot{v}_i = u_i, \quad (1)$$

where x_i is the position $[m]$, v_i $[m/s]$ is the speed, and u_i is the acceleration (control input).

We consider a main and a secondary road merging together (Fig. 1). A centralized controller derives the optimal control policy (acceleration/deceleration profile) in terms of fuel consumption for each vehicle cruising inside a particular radius—defined as the *control zone*—under the hard constraint to enable the vehicles to cross the *merging zone* without collision. The implicit assumption here is that each vehicle can communicate with the centralized controller and can transmit information regarding their locations and distances from the merging zone.

The optimal control policy of the centralized controller for each vehicle is communicated to the corresponding vehicle. If the vehicles are autonomous, then they will just follow the policy imposed by the controller. If there is a driver, however, then we assume that the driver will follow the control policy precisely—provided as instructions—of the centralized controller. Future research should investigate how to incentivize, or reinforce, drivers to follow these instructions.

We seek to optimize fuel consumption while improving the traffic flow on a merging point of two roads by coordinating the vehicles inside a *control zone*. We use the polynomial metamodel proposed in [25] that yields vehicle fuel consumption as a function of the speed and acceleration:

$$\dot{f}_v = \dot{f}_{cruise} + \dot{f}_{accel}, \quad (2)$$

where $\dot{f}_{cruise} = w_0 + w_1 \cdot v + w_2 \cdot v^2 + w_3 \cdot v^3$ estimates the fuel consumed by a vehicle traveling at a constant speed v, and $\dot{f}_{accel} = a \cdot (r_0 + r_1 \cdot v + r_2 \cdot v^2)$ is the additional fuel consumption when the vehicle accelerates with α. The polynomial coefficients w_j, $j= 0,\ldots,3$, and r_k, $k=0,1,2$, are calculated from experimental data. For the vehicle parameters reported in [25], where the vehicle mass is $M_v = 1,200$ kg, the drag coefficient is $C_D = 0.32$, the air density is $\rho_a = 1.184$ km/m^3, the frontal area is $A_f = 2.5$ m^2, and the rolling resistance coefficient is $\mu = 0.015$, the polynomial coefficients are equal to: $w_0 = 0.1569$, $w_1 = 2.45 \times 10^{-2}$, $w_2 = -7.415 \times 10^{-4}$, $w_3 = 5.975 \times 10^{-5}$, $r_0 = 0.07224$, $r_1 = 9.681 \times 10^{-2}$, and $r_2 = 1.075 \times 10^{-3}$.
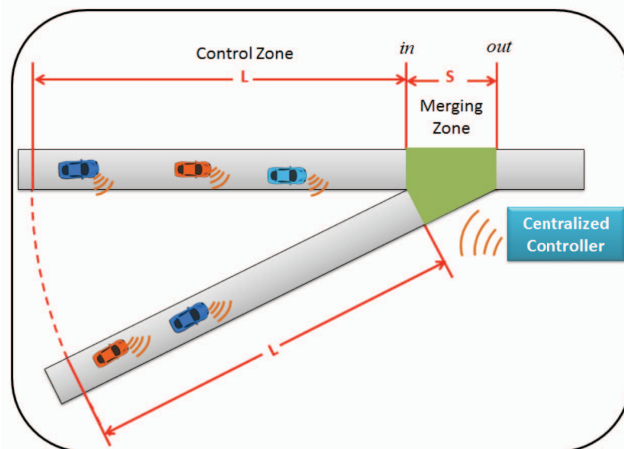

Fig. 1. Two one-lane merging roads.

Fig. 2 illustrates the fuel consumption variation with respect to the vehicle speed and acceleration. Apparently, there is a monotonic relationship between fuel consumption and acceleration. Consequently, instead of formulating a fuel consumption minimization problem we can formulate the problem considering directly vehicle acceleration in the objective function. In this context, the objective is to find for each vehicle the optimal acceleration profile from the time they enter in the *control zone* until the time the exit the *merging zone*.

III. SOLUTION APPROACH

To address this problem we consider the following three steps: (1) defining a hierarchical vehicle sequence based on which vehicle is closer to the *merging zone*, (2) assigning the times for each vehicle to reach and leave the *merging zone*

that guarantee collision avoidance, and (3) finding the closed-form analytical solution for the optimization problem.
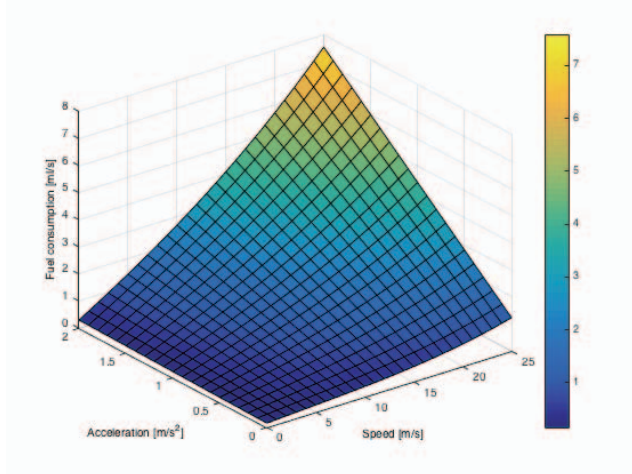

Fig. 2. Fuel consumption model.

1) Defining the hierarchical vehicle sequence

When a vehicle reaches the *control zone* it starts communicating its position to the centralized controller. Then the controller defines a hierarchical vehicle sequence starting with the vehicle that is closer to the *merging zone* (Fig. 3). If two vehicles on different roads have the same distance from the *merging zone*, the priority will be given to the vehicle on the main road. Note that with such a hierarchy, the problem of blocked lanes is avoided because, at each instant of time, only the vehicle that is closest to the *merging zone* will have the right-of-way. Furthermore, by imposing such hierarachy the problem can be easily extended to merging roads of multiple lanes.

In our analysis, we use a single subscript identifying each vehicle on the *control zone*, starting from the one that is closest to the *merging zone*, i.e., $i = 1$, to the one which is farthest to the *merging zone*.
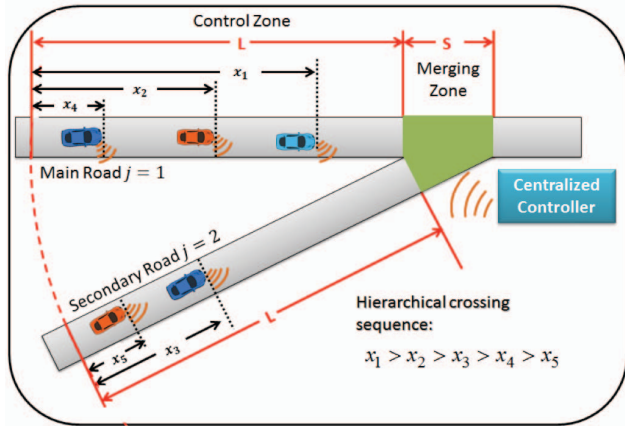

Fig. 3. Hierarchical crossing sequence.

2) Assigning the times to enter and exit the merging zone

Once the hierarchy is defined, the controller assigns to each vehicle i in the *control zone* the time, t_i^{in}, to enter the *merging zone*. To eliminate the chance of lateral collisions we impose the condition that only one vehicle at a time can be in the *merging zone*. Thus, the time for each vehicle i, t_i^{in}, to enter the *merging zone* is determined by the time, t_{i-1}^{out}, that the previous vehicle, $i-1$, in the hierarchy has exited it, as illustrated in Fig. 4. For vehicles traveling on the same road, this constraint is modified to maintain a minimum safe distance, δ, between them, denoted by t_{i+1}^{δ} as shown in Fig. 5.
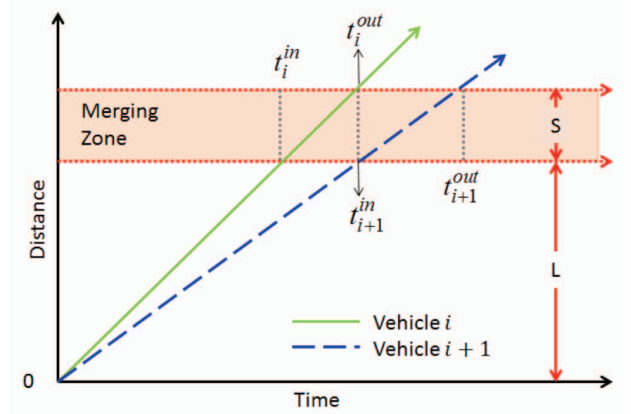

Fig. 4. Illustration of time calculation for vehicles entering the *merging zone* from different roads.
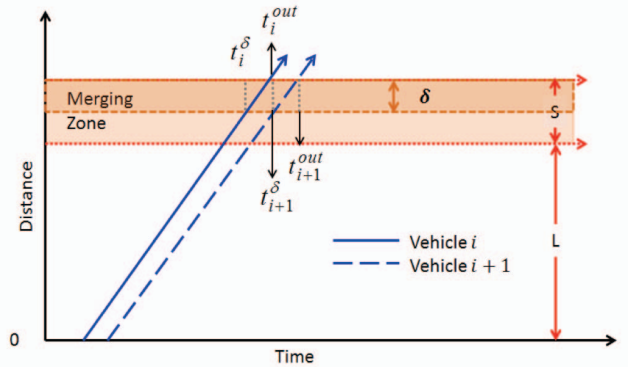

Fig. 5. Illustration of time calculation for vehicles entering the *merging zone* on the same road.

These time slots impose the constraints to avoid either lateral or rear end collisions and are assigned at each instant of time to allow readjustment according to the traffic conditions. Based on the previous two steps, the optimal control problem for n vehicles is formulated so as to minimize the L^2-norm of the control (acceleration/deceleration), namely

$$\min_{u_i} J = \min_{u_i} \frac{1}{2} \sum_{i=1}^{n} \int_0^{t_i^f} u_i^2 dt \qquad (3)$$

Subject to
- Vehicle dynamics:

$\dot{x}_i = v_i$

$\dot{v}_i = u_i$

- Initial conditions:

$x_i(t_i^0) = 0$

$v_i(t_i^0) = v_{des}$

- Final conditions:

$x_i(t_i^f) = L + S - x_i(t)$

$v_i(t_i^f) = v_{des}$

- Safety constraints:
    - Rear end collisions avoidance:

    $t_{i+1}^\delta \geq t_i^{out}$

    - Lateral collisions avoidance:

    $t_{i+1}^{in} \geq t_i^{out}$

where t_i^0 is the time that the vehicle i enters the *control zone*, $x_i(t)$ is the distance at time t that the vehicle i has proceeded inside the *control zone*, and t_i^f is the time the vehicle i exits the *merging zone*. Thus, the safety constraints have been translated to time constraints and will be used to the boundary conditions for the analytical solution. Since the initial vehicle speed when the vehicle enters the *control zone* is the driver's desired speed, we designate the final speed, when the vehicle exits the *merging zone* to be equal to the initial speed. However, this could be modified appropriately.

For the analytical solution of problem (3), we apply Pontryagin's minimum principle. We seek to find the optimal control $u^*(t)$ which drives the system along an optimal trayectory $x^*(t)$. For each vehicle i, the Hamiltonian function of the above optimization problem is

$$H_i(\lambda_i^x, \lambda_i^v, x_i, v_i) = \frac{1}{2} u_i^2 + \lambda_i^x v_i + \lambda_i^v u_i \qquad (4)$$

where λ_i^x and λ_i^v are the co-state components. Applying the Hamiltonian minimization condition, the optimal control can be given as a function of the co-states

$$u_i^* + \lambda_i^v = 0. \qquad (5)$$

The adjoin equations yield

$$\dot{\lambda}_i^x = -\frac{\partial H}{\partial x_i} = 0 \qquad (6)$$

$$\dot{\lambda}_i^v = -\frac{\partial H}{\partial v_i} = -\lambda_i^x, \qquad (7)$$

and hence $u_i^* = -\lambda_i^v$. From (6) we have $\lambda_i^x = a_i$ and from (7) implies $\lambda_i^v = -(a_i t + b_i)$, where a_i and b_i are constants of integration corresponding to each vehicle i. Consequently, the optimal control input (acceleration/deceleration) as a function of time is given by

$$u_i^*(t) = a_i t + b_i. \qquad (8)$$

Substituting the last equation to the vehicle dynamics equations (1) we can find the optimal speed and position for each vehicle, namely

$$v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i \qquad (9)$$

$$x_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i, \qquad (10)$$

where c_i and d_i are constants of integration. The constants of integration can be computed by the initial and final conditions in (3). Hence they are functions of time and state (position), i.e., $a_i(t, x_i), b_i(t, x_i), c_i(t, x_i)$, and $d_i(t, x_i)$. It is important to emphasize that this analytical solution can be implemented online. To derive online the optimal control for each vehicle, we need to update the integration constants at each time t. Equations (9) and (10) along with the initial and final conditions defined in the optimization problem (3) can be used to form a system of four equations of the form $\mathbf{T_i b_i = q_i}$. In this step, we are already satisfying the initial and final conditions, including the safety constraints.

$$\begin{bmatrix} \frac{1}{6}(t_i^0)^3 & \frac{1}{2}(t_i^0)^2 & t_i^0 & 1 \\ \frac{1}{2}(t_i^0)^2 & t_i^0 & 1 & 0 \\ \frac{1}{6}(t_i^f)^3 & \frac{1}{2}(t_i^f)^2 & t_i^f & 1 \\ \frac{1}{2}(t_i^f)^2 & t_i^f & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_i(t,x_i) \\ b_i(t,x_i) \\ c_i(t,x_i) \\ d_i(t,x_i) \end{bmatrix} = \begin{bmatrix} x_i(t_i^0) \\ v_i(t_i^0) \\ x_i(t_i^f) \\ v_i(t_i^f) \end{bmatrix} \quad (11)$$

Hence we have

$$\mathbf{b_i}(t, x_i(t)) = \mathbf{(T_i)^{-1}} \cdot \mathbf{q_i}(t, x_i(t)) \qquad (12)$$

where $\mathbf{b_i}(t, x_i(t))$ is a vector containing the four integration constants $a_i(t, x_i), b_i(t, x_i), c_i(t, x_i)$, and $d_i(t, x_i)$. Thus (8) can be written as

$$u_i^*(t, x_i) = a_i(t, x_i) \cdot t + b_i(t, x_i). \qquad (13)$$

Since (12) can be computed online the controller can yield the optimal control online for each vehicle i from (13).

IV. SIMULATION RESULTS

To validate the effectiveness of the efficiency of our analytical solution we simulated the merging scenario presented in previous section in Matlab/Simulink. In our case study, the length of the *control zone* is 400 m, and the *merging zone* length is 30 m. It is assumed that each vehicle travels at a constant speed of 30 mph (13.41 m/s) before entering the *control zone*. As soon as a vehicle reaches the *control zone* then the centralized controller designates the acceleration/deceleration profile for each vehicle until it exits the *merging zone*. All vehicles are assumed to have the characteristics described in Section II.

We considered the case of coordinating 30 vehicles, 15 for each road. The centralized controller is able to derive online the optimal control policy (acceleration/deceleration profile) in terms of fuel consumption by avoiding collision in the *merging zone*, while only one vehicle at the time was crossing the *merging zone*, as illustrated in Fig. 6. We note that as the number of vehicles on each road in the *control zone* increases, there is an impact on the acceleration profile for each vehicle (Fig. 7). The controller accelerates the vehicles that are closer to the *merging zone* to create more space in the road for the vehicles following. However, as the number of vehicles on the road increases and reaches its maximum capacity, eventually the vehicles entering the *control zone* will need to decelerate or even come to a full stop as imposed by the road capacity constraints. As a result, the vehicles ahead in the hierarchy are able to cross the *control zone* in a shorter time than the rest of the vehicles.
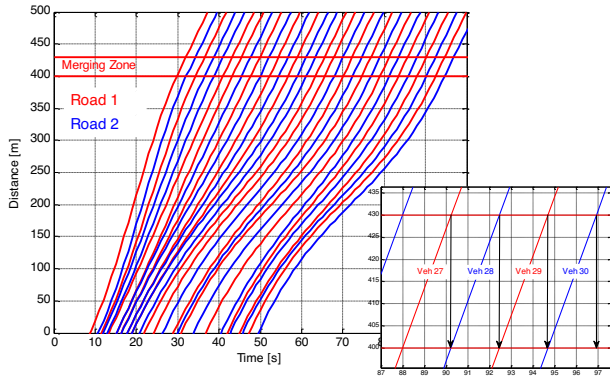

Fig. 6. Distance of the thirty vehicles traveled in merging coordination (road 1 corresponds to the main road and road 2 corresponds to the secondary road).

The optimal solution for the vehicle coordination was compared to a baseline scenario. In the baseline scenario, the vehicles on the main road have the right-of-way. Thus all the vehicles in the secondary road need to come to a full stop, before they enter the *merging zone,* and wait until the vehicles

on the main road cross the *merging zone*. The optimal acceleration/deceleration profile imposed by the controller resulted in minimizing fuel consumption both at the *control zone* and *merging zone* as shown in Fig. 8. The fuel consumption improvement at the *merging zone* is due to the fact that the vehicles coming from the secondary road do not come to a full stop before they enter to the main road, thereby conserving momentum and fuel while also improving travel time. The overall fuel consumption improvement when the vehicles are coordinating compared to the baseline scenario is 49.8%. Moreover, the coordination of vehicles resulted in improving the total travel time by 6.9% compared to the baseline (Fig. 9).
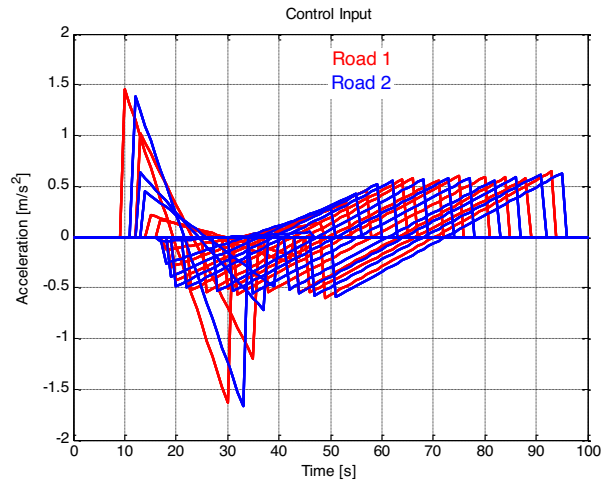

Fig. 7. Acceleration profile in merging coordination of thirty vehicles (road 1 corresponds to the main road and road 2 corresponds to the secondary road).
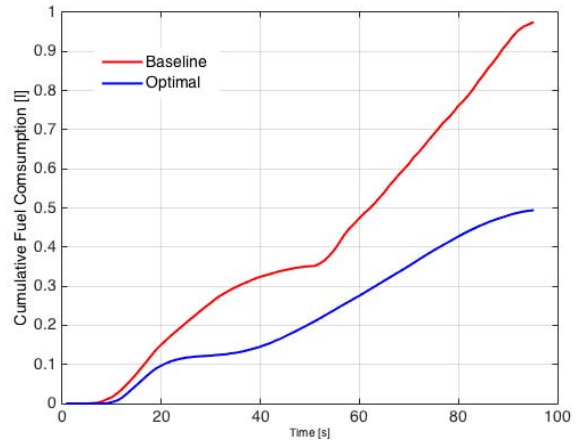

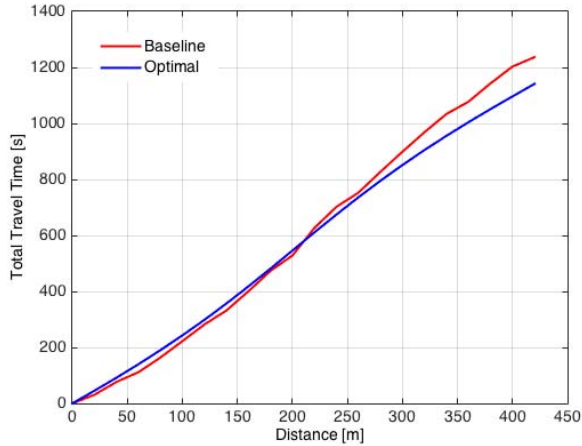Fig. 8. Cumulative fuel consumption comparison.

Fig. 9. Total travel time.

V. Concluding Remarks

In this paper, we developed an analytical solution for the problem of coordinating online connected vehicles traveling over two merging roads. The problem was formulated as a constrained optimal control problem that aims to find a safe and fuel-efficient crossing schedule, i.e., velocity profile and arrival time to the *merging zone*, for all the vehicles in a *control zone*. Fuel-efficiency was addressed by using acceleration as the objective function of the optimal control problem. We applied Pontryagin's minimum principle and showed that it is possible to obtain a closed-form solution, which allows the implementation of an online optimal control policy. The efficiency of the effectiveness of the optimal control policy was validated through simulation and compared to a baseline scenario, where all the vehicles in the secondary road need to come to a full stop, before they enter the *merging zone,* and wait until the vehicles on the main road cross the *merging zone*.

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