

# Application of Optimal Production Control theory for Home Energy Management in a Micro Grid

Jin Dong, Andreas A. Malikopoulos, Seddik M. Djouadi and Teja Kuruganti

**Abstract**—We consider the optimal stochastic control problem for home energy systems with solar and energy storage devices when the demand is realized from the grid. The demand is subject to Brownian motions with both drift and variance parameters modulated by a continuous-time Markov chain that represents the regime of electricity price. We model the systems as pure stochastic differential equation models, and then we follow the completing square technique to solve the stochastic home energy management problem. The effectiveness of the efficiency of the proposed approach is validated through a simulation example. For practical situations with constraints consistent to those studied here, our results imply the proposed framework could reduce the electricity cost from short-term purchase in peak hour market.

## I. INTRODUCTION

Several efforts have been utilizing clean renewable energy sources and demand-side resources, e.g., electric loads, to reduce green house gas emissions of the electric power grid. However, there are several challenges associated with the uncertainty of renewable generation and inelastic demand. Furthermore, the interdependencies between system states of power networks or interconnected loads complicate the decision-making process. Growing interactions between power and energy systems, and human agents with advances in sensing, computing and communication technologies has also increased the need for personalized operations. Consumers can optimally adjust their energy consumptions by participating into the demand response (DR) program for minimizing the electricity bill [1], [2].

In this paper, we consider the following problem: given a stochastic renewable generation, which is disclosed in time, a customer agent seeks to conduct an efficient demand response, and then allocate the net remaining generation among various conventional generators.

Recently, system operators have turned to demand-side resources to mitigate peak demand. The market is set up as

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a two-settlement electricity market, commonly used in U.S. independent system operator/regional transmission operator (ISO/RTO) markets [3]. In a typical two-settlement electricity market, load entities can procure energy from a day-ahead market and a real-time market [4]. In order to reduce the expansive real-time market purchase during peak hours, two general solution approaches have been used in the literature: 1) utilizing time-shiftable loads and 2) optimal demand bidding.

Model predictive control and dynamic programming (DP) are the two widely adopted techniques for solving these problems utilizing time-shiftable loads. The problem of scheduling time-shiftable loads under different retail electricity pricing scenarios has been addressed in [5], [6], [7] for thermal loads, in [8], [9] for hybrid electric vehicles, in [10], [11] for aggregated loads, while several other efforts are based on DP [12], [13]. Optimal demand bidding has been discussed for fixed or forecasted load data in [14], [15], and for time-shiftable loads with deadlines in [9].

Furthermore, due to the nature of the smart grid which features different entities with conflicting objectives, game theoretical approaches have been proposed in a number of papers [16], [17], [18]. Zhu et al. [19] used a Stackelberg game framework for economic dispatch with demand response, where they used a two person game with ISO as leader and users aggregated into second player. The users could change their demand based on price signal so as to maximize their payoff function. In problems involving systems with a large state space, where using dynamic programming is computationally intense, approximate DP has been considered as a viable alternative (see [20]).

In this paper, we model the users' demand and day-ahead market bidding as stochastic dynamical systems with regime switching and generate the optimal demand response in a stochastic way. We use the optimal management technique (specially the completing square) to solve the problem in the real-time case.

## A. Contribution

By considering the optimal production control framework [21], there are two main advantages:

1. Home energy management is made feasible for any existing power grid without the requirement to have detailed electricity price available to the customers.
2. Better performance on the infrastructure where electricity price is available to the customers can be achieved since real stochastic models are used to capture the uncertainty in the power systems especially for the renewable energy penetrated situation.

The paper is organized as follows. We present the modeling framework and problem formulation in Section II, and the main results in Section III. Then, we provide an illustrative example in Section IV, and draw our concluding remarks in Section V.

## II. PRELIMINARIES: NOTATION AND PROBLEM FORMULATION

We assume that the cumulative demand of a customer follows a Brownian motion with drift modulated by a continuous-time Markov chain that alternates between two regimes. We allow both the drift and the diffusion to be affected by the Markov chain, which represents the regime (or the state) of the economy. One regime may represent an off-peak period with a low demand rate and the other may represent a peak period with a high demand rate. The objective of management is to maintain the battery close to a fixed target level; there is a penalty associated with the deviation of the battery's state of charge (SOC) from its target value. In addition, the customer may also prefer to maintain a production rate that is close to a fixed target rate to obtain the best efficiency of the generator. There is also a running cost associated with the difference between the actual production rate and its target. In this paper we consider two basic models.

In the first model, management knows at any time the actual regime of the electricity price, whereas in the second model we assume that management does not know the actual regime. We determine for both models the optimal production policies by applying the "completing squares" technique.

### A. Notation

Throughout the paper  $\omega, \omega_i$  etc. will be used to denote random variables. We denote random variables with upper case letters, and their realization with lower case letters, e.g., for a random variable  $X$ ,  $x$  denotes its realization. We assume  $X$  and  $D$  are adapted stochastic processes defined as the following notation:

TABLE II.1  
TABLE OF NOTATION

$X_t$	$\triangleq$	battery state of charge (SOC) at time $t$ ,
$G_t$	$\triangleq$	amount (Kwh) purchased (sold) from (to) grid at time $t$ ,
$Y_t$	$\triangleq$	cumulative PV generation (Kwh) up to time $t$ ,
$D_t$	$\triangleq$	cumulative demand (Kwh) up to time $t$ ,
$p_t$	$\triangleq$	diesel generator production rate (Kw) at time $t$ ,
$\epsilon(t)$	$\triangleq$	regime of the electricity price at time $t$ .
Regime switching variables of two-state model		
$\epsilon(t)$	$=$	$\begin{cases} 1, & \text{electricity is in off-peak price} \\ 2, & \text{electricity is in peak price} \end{cases}$

We consider a continuous-time Markov chain  $\{\epsilon(t), t \geq 0\}$ , that can take values in two regimes  $\varphi = \{1, 2\}$ . The amount of time that the electricity price remains in the off-peak (or peak) regime is exponentially distributed with rate

$\lambda_1(\lambda_2)$ . Depending on whether the electricity price is *off-peak* (*peak*), i.e., belonging to regime 1(2), the cumulative consumer demand follows a Brownian motion  $\omega$  with a drift  $\mu_1(\mu_2)$  and a variance  $\sigma_1^2(\sigma_2^2)$ . We also assume that  $\epsilon$  and  $\omega$  are independent. Therefore, the Markov chain  $\epsilon$  has a strongly irreducible generator  $Q = [v_{ij}]_{2 \times 2}$ , for  $i, j \in \varphi$ , where  $v_{ij} = -\lambda_i < 0$  and  $\sum_{j \in \varphi} v_{ij} = 0$  for every  $i \in \varphi$ .

More generally, one can generalize the state space to  $\varphi = \{0, 1, 2, \dots, N, \}$  for  $\epsilon(t)$  to model more complex information structures in the future.

We should notice that the cumulative demand  $D_t$  may at times decrease in  $t$ . This situation would correspond to a potential negative demand, i.e., electric may be sold back to the grid when sufficient power supplied from the renewable resources and the battery storage is full.

Table II.1 summarizes the notation used in this paper. Note when  $\epsilon(t) = 1$ , we represents a regime of normal electricity price, while when  $\epsilon(t) = 2$  we represent a regime of peak electricity price. Apparently,  $\epsilon$  could also represent the cases where consumer demand is high and low.

### B. Model and problem formulation

The dynamic system can be modeled as a controlled regime-switching diffusion. The switching process reflects system structural changes, exemplified by scheduled or emergency maintenance of solar modules, e.g., failure of a battery cell, addition of super-capacitor banks and tap changes in transformer. We consider the regime switching to be the electricity price.

To meet the total power consumption demand, it is necessary to impose certain constraints. On the other hand, to maintain grid functionality, smooth operations and reduce waste, it is desirable that (generation - consumption) disparity in transient be kept as small as possible. The problem is thus naturally formulated as a mean-variance control problem. The objective is to minimize the risk (measured by variance) of the terminal battery storage subject to a given expected terminal storage level.

1) *Load (demand) model:* We consider that the *cumulative demand* satisfies the dynamics

$$dD_t = \mu_{\epsilon(t)} dt + \sigma_{\epsilon(t)} d\omega_t, \quad (\text{II.1})$$

where at any time  $t$ , the demand rate  $\mu_{\epsilon(t)}$  and the diffusion  $\sigma_{\epsilon(t)}$  depend on the regime  $\epsilon(t)$ ,  $d\omega(t)$  represents the Brownian Motion.

*Remark 2.1:* We consider the loads are uncontrollable. The solution of the problem with controllable loads, e.g., thermostatically controlled loads, is the subject of ongoing research.

2) *Photo-voltaic model:* The output of photo-voltaic (PV) is associated with significant uncertainty. On the other hand, a renewable generator's maximum capacity is a stochastic process, e.g., the wind turbine's maximum power is determined by the wind speed and direction. Similarly, a PV system's output is determined by how much solar radiation is available

at a given time, weather condition, and the angle that the sunlight is shining on the solar panels.

$$dY_t = \mu_{p(\epsilon(t))}dt + \sigma_{p(\epsilon(t))}d\omega_t, \quad (\text{II.2})$$

where the *drift term*  $\mu_{p(\epsilon(t))}$  represents *average* solar radiation values for each time step (the smooth curve for maximum output); and the *diffusion term*  $\sigma_{p(\epsilon(t))}$  represents solar radiation *fluctuations* which are commonly caused by many factors such as wind, clouds, many other uncertain weather conditions.

3) *Diesel generator*: Backup diesel generators can supply load requirements that exceed renewable generation to increase the reliability of power systems. Besides meeting demand directly, diesel generators can also be used for battery charging, especially when it is during the peak hours. As a device used to convert the energy stored in diesel fuel into the electrical energy used by household and industrial devices, fuel consumption is the major portion of the operating cost.

4) *Power balance*: In order to achieve the energy management, it is necessary to satisfy the power balance in the grid. Therefore, the battery storage level  $X$  has to satisfy the following equation with notations defined in Table II.1

$$X_t = x_0 + \int_0^t p_s ds - D_t + Y_t + G_t \quad (\text{II.3})$$

where  $X_t$  can be computed by

$$\begin{aligned} X_t &= x_0 + \int_0^t (p_s - \mu_\epsilon(s) + \mu_{p(\epsilon(s))}) ds \\ &\quad - \int_0^t (\sigma_\epsilon(s) + \sigma_{p(\epsilon(s))}) d\omega_s + G_t \\ &= x_0 + \int_0^t (p_s - \underbrace{(\mu_\epsilon(s) - \mu_{p(\epsilon(s))})}_{\mu_{\lambda(\epsilon(s))}}) ds \\ &\quad - \int_0^t \underbrace{(\sigma_\epsilon(s) + \sigma_{p(\epsilon(s))})}_{\sigma_{\lambda(\epsilon(s))}} d\omega_s + G_t. \end{aligned} \quad (\text{II.4})$$

Through this convention, we could potentially generalize to larger systems that could involve more components. In this case, since we can describe each component in a stochastic way, we would include corresponding terms to get an augmented formulation as shown in the last line of (II.4). Note that we allow the demand to take *negative values*, which represent the abundant electricity sells back to the grid when we have more renewable generation than demand. The system structure is shown in Fig. II.1.

*Remark 2.2*: By allowing regime switching in the models complicates the problem significantly since it no longer follow a standard Brownian motion with drift. However, there are practical cases in which it is more realistic to assume that the demand rate is a modulated *geometric Brownian Motion*.

The aggregator operates the power grid and aims to minimize the long-term time-averaged system cost by jointly managing supply, demand, and storage units. With increased integration of renewable generation and energy storage, business models of power system operators and electricity

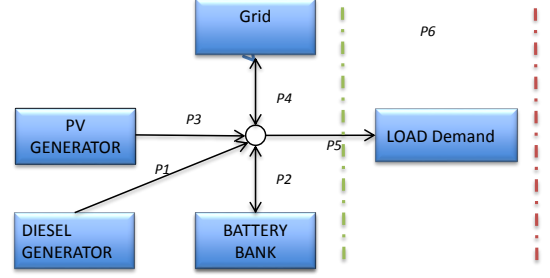


Fig. II.1. Infrastructure with uncontrollable loads only.

markets are constantly evolving. From the study in [22], one suggested model of future electric utilities is termed as "energy services utility." If we consider each residential home user as an individual agent or utility, we can formulate our problem as follows.

*Problem 2.3*: Each agent seeks to select an optimal production rate  $p : [0, \infty) \times \Omega \mapsto (-\infty, \infty)$  for the local generators that minimizes the functional  $J$

$$\begin{aligned} J(p) &= E \left[ \int_0^\infty R_t \{ \alpha_{\epsilon(t)} (X_t - \mathcal{S}_{\epsilon(t)})^2 \right. \\ &\quad \left. + \beta_{\epsilon(t)} (p_t - \mathcal{P}_{\epsilon(t)})^2 \} + G_t^2 + p_t^2 dt \right], \end{aligned} \quad (\text{II.5})$$

where

$$R_t = \exp \left\{ - \int_0^t r_{\epsilon(u)} du \right\},$$

and  $r_i \in (0, \infty)$ ,  $\alpha_i \in (0, \infty)$ ,  $\beta_i \in (0, \infty)$ ,  $\mathcal{S}_i \in (-\infty, \infty)$ ,  $\mathcal{P}_i \in (-\infty, \infty)$  are constants for each regime  $\epsilon(t) = i$  ( $i \in \{1, 2\}$ ).

The cost functional  $J(p)$  represents not only the cost from diesel generator and grid but also the running cost incurred by deviating from the target battery SOC  $\mathcal{S}_i$  and from the diesel generator production target rate  $\mathcal{P}_i$ . Both of the target rates  $\mathcal{S}_i$  and  $\mathcal{P}_i$  help the battery and diesel generator pick the most effective working strategy according to the regime of electricity price. In specific, SOC is an indicator for battery storage [23]. An appropriate range of the SOC of the battery should be guaranteed to prevent the battery from being over- or under-charged. For example, when the electricity is in peak price, the battery may pick a small value of  $\mathcal{S}_i$  sacrifice to the life-cycle by almost depleting itself. In contrast, it would pick a large value to store as much as possible when the electricity price is low.

*Remark 2.4*: We should emphasize that we seek to minimize the diesel generator production. Therefore the production target rate  $\mathcal{P}_i$  is ideally set to 0 all the time. For a more general framework, we could take advantage of our switching framework by setting it as:

$$\mathcal{P}_i = \begin{cases} Pr_{min}, & \text{if } \epsilon(t) = 1 \text{ (off-peak price),} \\ Pr_{max}, & \text{if } \epsilon(t) = 2 \text{ (peak price),} \end{cases}$$

where  $Pr_{min}$  and  $Pr_{max}$  represent the minimum and maximum production rate of the range with the best efficiency.

In the market with aggregator involved, most of the trading decisions are made ahead of time. Then our local components (battery, diesel generator and PV panel) are working as supplementary power sources to supply the demand fluctuations. For simplicity, we will be focusing on the case with fixed 24-hour ahead bid strategy such that the consumer has made the decision for purchasing how much electricity in advance. By observing a large historical data of the purchase for each time period, we could fit the trade transaction into a stochastic model similar as load and PV models:

$$dG_t = \mu_{G(\epsilon(t))}dt + \sigma_{G(\epsilon(t))}d\omega_t. \quad (\text{II.6})$$

Then we could plug it into (II.4) to rewrite it as:

$$\begin{aligned} X_t &= x_0 + \int_0^t (p_s - \underbrace{(\mu_\epsilon(s) - \mu_{p(\epsilon(s))} - \mu_{G(\epsilon(s))})}_{\mu_{\lambda(\epsilon(s))}})ds \\ &\quad - \int_0^t \underbrace{(\sigma_\epsilon(s) + \sigma_{p(\epsilon(s))} + \sigma_{G(\epsilon(s))})}_{\sigma_{\lambda(\epsilon(s))}} d\omega_s. \end{aligned} \quad (\text{II.7})$$

Therefore, the cost function would be rewritten as:

$$J(p) := E \left[ \int_0^\infty R_t \{ \alpha_{\epsilon(t)} (X_t - \mathcal{S}_{\epsilon(t)})^2 + \beta_\epsilon(t) p_t^2 \} dt \right]. \quad (\text{II.8})$$

### III. MAIN RESULTS

Following the models and formulation introduced in the last section, we will present the main results of this paper which is to derive an optimal production rule for the stochastic home energy management problem.

#### A. With regime known

In this section we will discuss the case in which the regime is known to each customer (agent). This is typically possible in the modern electricity market where the ISO keeps broadcasting the current price to the users. Then the current regime will be available to the customers following the policy defined before ( $\epsilon(t) = 1$  could represent a regime of normal electricity price while  $\epsilon(t) = 2$  could represent a regime of peak electricity price).

*Assumption 3.1:* There exists modern communication structure in the micro grid to broadcast the electricity price to each customer.

Let  $\{\mathcal{F}_t^{(W, \epsilon)}\}$  denotes the  $\sigma$ -algebra generated by Brownian motions  $W(s)$  and  $\epsilon(s)$ ,  $t \in [0, \infty)$ . Intuitively  $\mathcal{F}_t$  contains all the information up to time  $t$ .

*Definition 3.2:* A production process  $p$  is admissible if it is adapted to the filtration  $\{\mathcal{F}_t; t \in [0, \infty)\}$ , and the corresponding inventory process satisfies

$$\int_0^\infty E[R_t X_t^2] dt < \infty \quad \text{and} \quad \lim_{t \rightarrow \infty} E[R_t X_t] = 0. \quad (\text{III.1})$$

The first inequality in (III.1) implies

$$\liminf_{t \rightarrow \infty} E[R_t X_t^2] = 0. \quad (\text{III.2})$$

The objective is to minimize the cost functional  $V_f = \inf_{p \in \mathcal{A}} J(p)$ , where  $\mathcal{A}$  denotes the class of admissible production processes.

If we follow the procedure to define necessary constants, we are guaranteed to obtain the complete square with all the other terms vanished.

*Theorem 3.3:* [21] The optimal production rate  $p^{(f)}$  is given in feedback form as:

$$p_s^{(f)} = -\frac{a_{\epsilon(s)}}{\beta_{\epsilon(s)}} X_s^{(f)} - \frac{f_{\epsilon(s)}}{2\beta_{\epsilon(s)}}, \quad (\text{III.3})$$

where  $X^{(f)}$  is the battery storage process corresponding to the above production process and  $a_i$  is the unique positive solution of

$$-r_i a_i + \alpha_i - \frac{1}{\beta} a_i^2 + \lambda_i a_{3-i} - \lambda_i a_i = 0, \quad (\text{III.4})$$

and  $f_i$  is defined as:

$$\begin{aligned} f_i &= \frac{2}{\gamma} \left\{ [-a_i \mu_i - \alpha_i \mathcal{S}_i] \left[ \frac{a_{3-i}}{\beta_{3-i}} + r_{3-i} + \lambda_{3-i} \right] \right. \\ &\quad \left. - \lambda_i a_{3-i} \mu_{3-i} - \lambda_i \alpha_{3-i} \mathcal{S}_{3-i} \right\}, \end{aligned} \quad (\text{III.5})$$

where

$$\gamma = \left( \frac{a_1}{\beta_1} + r_1 + \lambda_1 \right) \left( \frac{a_2}{\beta_2} + r_2 + \lambda_2 \right) - \lambda_1 \lambda_2. \quad (\text{III.6})$$

The index  $f$  means full information here since we know the regime.

*Proof:* The proof has been adapted the *completing squares method* presented in [21]. Space limitations do not permit us to include the details.  $\blacksquare$

#### B. With regime unknown

In this section, we generalize the problem by removing the assumption in Theorem 3.3. This is a more general infrastructure of our power grid (no communication from the ISO), where each individual residential customer has few information about how his/her neighbors are using. Therefore, each agent doesn't have exact information about whether they are in normal or peak hours except by experience. Again, this experience knowledge might involve a lot of bias or errors due to the limited information here.

Under this situation, the regime state  $\epsilon(t)$  is unknown to us, we could only observe the demand  $D_t$  here.

However, it turns out that the problem is tractable even if the targets are still regime dependent (even though the states are unknown to the customer). We seek to optimize the local production rate by inferring the correct regime state first.

Let  $\{\mathcal{F}_t^D, t < \infty\}$  be the filtration generated by the demand process  $\{D_t, t < \infty\}$ . We now require that the production process  $p$  be adapted to the filtration  $\{\mathcal{F}_t^D, t < \infty\}$ .

1) *Optimal production by completing square*: Considering no regime information available, the parameters in the original problem should be modified with

$$r_1 = r_2 = r, \quad \sigma_1^2 = \sigma_2^2 = \sigma, \quad \alpha_1 = \alpha_2 = \alpha,$$

and  $\beta_1 = \beta_2 = \beta$ .

Also we have  $a_1 = a_2 = a$ , and  $a$  is the positive root of the equation

$$-ra + \alpha - \frac{1}{\beta}a^2 = 0. \quad (\text{III.7})$$

Let  $\mathcal{A}_l$  be the class of admissible production processes (the subscript  $l$  refers to the limited information). The cost associated with a production process  $p$  becomes:

$$J(p) = E \left[ \int_0^\infty R_t \{ \alpha (X_t - \mathcal{L}_{\epsilon(t)})^2 + \beta p_t^2 \} dt \right], \quad (\text{III.8})$$

where now  $R_t = e^{-rt}$ . The minimal cost is  $V_l = \inf_{p \in \mathcal{A}_l} J(p)$ .

If we follow the procedure to define necessary constants, we are guaranteed to obtain the complete square with all the other terms vanished.

*Theorem 3.4*: [21] The optimal production rate  $p^{(l)}$  with limited information regarding to the state of the regime is given by:

$$p_s^{(l)} = -\frac{\alpha}{\beta} X_s^{(l)} + E \left[ -\frac{1}{2\beta} f_{\epsilon(s)} | \mathcal{F}_s^D \right], \quad (\text{III.9})$$

which can be written as

$$p_s^{(l)} = -\frac{\alpha}{\beta} X_s^{(l)} - \frac{1}{2\beta} f_1 P \{ \epsilon(s) = 1 | \mathcal{F}_s^D \} - \frac{1}{2\beta} f_2 P \{ \epsilon(s) = 2 | \mathcal{F}_s^D \}. \quad (\text{III.10})$$

where  $l$  denotes limited information.

*Proof*: Similarly as Theorem 3.3, the proof is based on the *completing squares method*, and we omit this part due to space limit. However, it should be emphasized that the expression for optimal production here is much more complicated than that of (III.3) due to the fact that we only have limited information available. In order to make (III.10) computable, we must calculate the conditional probabilities  $P \{ \epsilon(s) = 1 | \mathcal{F}_s^D \}$  and  $P \{ \epsilon(s) = 2 | \mathcal{F}_s^D \}$  first.

This objective can be achieved by utilizing the tools from optimal estimation for hidden Markov model (HMM) [24] if we denote the conditional probabilities as a two-dimensional process  $\epsilon(t)(t) = (\eta_1(t), \eta_2(t))$ . Then the actual regime state can be estimated by change of measure based on the *Girsanov's theorem*. Finally, the problem boils down solving a well-known *stochastic differential equation (SDE)*, which can be solved directly either in a explicit way or numerically. Due to the space limit here, this will be discussed in another forthcoming paper. ■

## IV. NUMERICAL RESULTS

In this section, we present a numerical example to demonstrate the effectiveness of the efficiency of the controller presented in this paper. For simplicity, we only consider the full information case, which corresponds to the cases with regime known and unknown, respectively. The system structure is the same as given in Fig. II.1 while the regime is available. The time intervals for peak hours are assumed to be "7 : 00 ~ 8 : 00", "12 : 00 ~ 13 : 00" and "18 : 00 ~ 20 : 00". So in total there are 4 hours peak-time during a day.

*Example 4.1*: Consider demand drifts  $\mu_1 = 0.5$ ,  $\mu_2 = 3.0$ ;  $\mu_{p1} = 0.1$ ,  $\mu_{p2} = 1.5$ ;  $\mu_{G1} = 0.3$ ,  $\mu_{G2} = 1.0$ . All  $\sigma_i$ 's are assigned to be 0.2. The expected time in regime 1 (off-peak) is 6 times than the expected amount of time in regime 2 (peak), such that  $\lambda_1 = 1.2$ ,  $\lambda_2 = 0.2$ .

$$r_1 = r_2 = 0.2, \quad \alpha_1 = \alpha_2 = 1.0, \quad \beta_1 = 2, \quad \beta_2 = 0.5, \quad (\text{IV.1})$$

$\mathcal{S}_1 = 0.9$  and  $\mathcal{S}_2 = 0.2$  which represent the target battery SOC for both off-peak and peak regimes, respectively.

Plug into (III.4), we get

$$\begin{aligned} -0.2 * a_1 + 1 - 0.5 * a_1^2 + 0.2 * a_2 - 0.2 * a_1 &= 0, \\ -0.2 * a_2 + 1 - 0.5 * a_2^2 + 0.2 * a_1 - 0.2 * a_2 &= 0. \end{aligned}$$

Then we obtain  $a_1 = a_2 = 2.1383$ .

We consider initial values of cumulative demand, diesel production rate and battery SOC to be 0, 0, and 0.8, respectively. Moreover, the initial regime is assumed to be regime 1 which means off-peak price. The 24-hours time horizon is discretized by partitioning it into 1,000 time intervals. Regime switchings occur at the instances marked as red diamond in Fig. IV.1 - IV.3.

A sample path of the *cumulative* demand is given in Fig. IV.1. It's worthy differentiating this cumulative demand up to time step  $t$  with the instant demand for each time step.

The control signal for the diesel generator is depicted in Fig. IV.2. Recall that we want to minimize the diesel cost which is equivalent to choose the control to be as close as 0 whenever it is not necessary to run the generator. Moreover, we do see the generator kicks in when the electricity is in the peak price. Since the cost to purchase extra electricity at this peak hours is usually extremely expansive, it is wiser to turn on the generator to meet the demand.

Finally, the dynamics of battery SOC is captured in Fig. IV.3. It is obvious that the battery keeps being fully charged whenever it's in off-peak hours, while it reaches the minimum SOC as in the peak hours.

## V. CONCLUSION

In this paper, we addressed the optimal stochastic control problem for home energy systems with solar and energy storage devices when the demand is realized from grid. The demand is subject to Brownian motions with both drift and variance parameters modulated by a continuous-time Markov chain that represents the regime of electricity price. We presented a framework aimed at addressing the stochastic

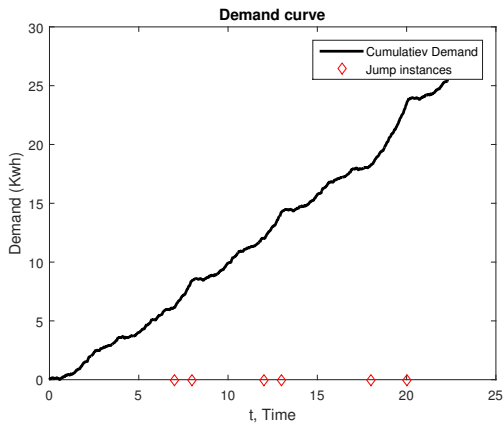


Fig. IV.1. Cumulative demand.

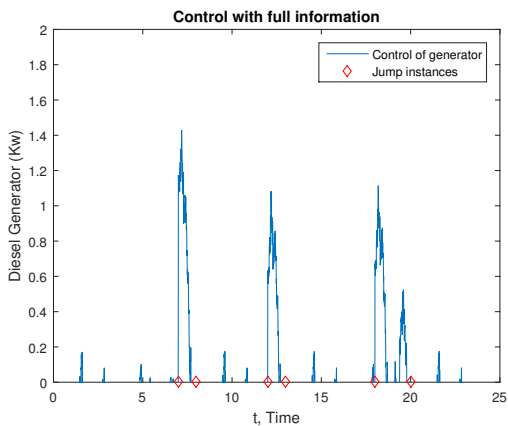


Fig. IV.2. Control for diesel generator.

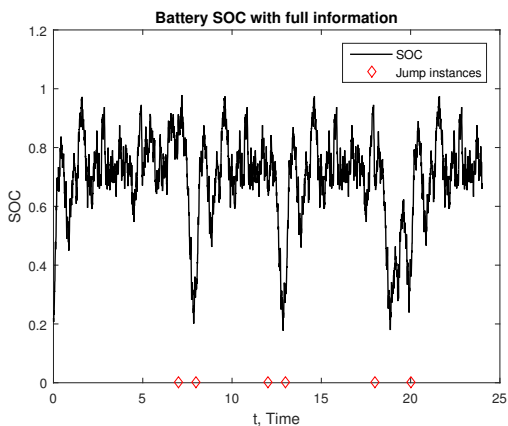


Fig. IV.3. SOC.

processes involved in home energy management systems, and used mean-variance to improve the efficiency of the power grid. Ongoing research includes detailed derivation for the optimal estimation of HMM and the corresponding simulation. Future research should investigate a comparison with traditional optimization framework.

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