Development and Evaluation of Speed Harmonization using Optimal Control Theory: A Simulation-Based Case Study at a Speed Reduction Zone

Seongah Hong (corresponding author)
Research Assistant
University of Virginia
351 McCormick Road
Charlottesville, VA 22904
Tel: 703-965-0244; Email: sh3zm@virginia.edu

Andreas A. Malikopoulos
Deputy Director, Urban Dynamics Institute
Energy and Transportation Science Division
Oak Ridge National Laboratory
Oak Ridge, TN 37831
Tel: 865-946-1529; Email: andreas@ornl.gov

Joyoung Lee
Assistant Professor
Department of Civil Engineering
New Jersey Institute of Technology
University Heights
Newark, NJ 07102
Tel: 434-806-3152; Email: jo.y.lee@njit.edu

Byungkyu Brian Park
Associate Professor
Department of Civil and Environmental Engineering
University of Virginia
P.O. Box 400742
Charlottesville, VA 22904-4742
Tel: 434-466-9001; Email: bpark@virginia.edu

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ABSTRACT

We address the problem of harmonizing the speed of an increasing number of vehicles on a highway in real time. The objective is to derive the optimal acceleration/deceleration of each vehicle that harmonizes the speed of an increasing number of vehicles at a speed reduction zone on the highway, under the hard safety constraint to avoid rear-end collision. We formulate the control problem and provide an analytical and closed-form solution that can be implemented in real time. The solution yields the optimal acceleration/deceleration of each vehicle under the hard constraint of collision avoidance at the speed reduction zone. The effectiveness of the solution is evaluated through simulation and it is shown that the proposed approach can reduce significantly both fuel consumption and travel time. For different traffic volume levels, the per-vehicle fuel consumption were reduced by 12-17% over the base case and 2-12% over the state-of-the-art VSL algorithm. The travel time was reduced by 28-32% over the base case and 11-28% over the VSL algorithm.

Keywords: Speed harmonization, connected and automated vehicles, traffic flow, optimal control, energy usage

I. INTRODUCTION

Motivation

In a rapidly urbanizing world, we need to make fundamental transformations in how we use and access transportation. This starts with the observation that the purpose of a transportation system is not mobility but rather accessibility to goods, services, and activities. Mobility is only an unintended outcome of our accessibility needs and may be viewed as an intermediate service (the means) on the way to what we really want: access. As we move to increasingly complex systems *(1)*, new control approaches are needed to optimize the impact on system behavior of the interplay between vehicles at different traffic scenarios.

Intersections, merging roadways *(*2, 3*)*, speed reduction zones along with the drivers' responses to various disturbances *(4)* are the primary sources of bottlenecks that contribute to traffic congestion *(5)*. In 2015, congestion caused people in urban areas to spend 6.9 billion hours more on the road and to purchase an extra 3.1 billion gallons of fuel, resulting in a total cost estimated at $160 billion *(6)*. In the US, the average hours annually wasted per commuter was estimated as 50 hours, having ranked the worst country worldwide *(7)*. Particularly, in the most congested metropolitan areas including Los Angeles, CA (81 hours), Washington DC (75 hours) and San Francisco, CA (75 hours), in which cities every driver has wasted more than three days in a gridlock traffic a year *(7)*.

Speed harmonization is one of the major Intelligent Transportation Systems (ITS) applications operated in the US. Instead of having drivers go high speed into a jam, the drivers approach slowly earlier in the upstream, the speed of queue build-up decreases, therefore the congestion recovery time is improved. Eventually, even though their speed may be temporarily reduced, the system is processing vehicles faster. The idea of speed harmonization has been realized through various techniques, such as Variable Message Signs (VMS), Variable Speed Limit (VSL) and the rolling speed harmonization (a.k.a., pace-car technique) *(8)*. Both VMS and VSL systems employ the display gantries mounted along roadways to deliver messages or control schemes. Typical messages provided through VMS systems are road/exit closures, crashes, maintenance/constructions, weather warnings, estimated travel times, etc. While, VSL provides the dynamic speed limits to traffic flow approaching the queues at the downstream bottleneck to reduce the speed variances and mitigate shock waves effects. Another strategy of speed harmonization is so-called the rolling speed harmonization. It uses the designated patrol vehicles entering the traffic to hold a traffic stream to follow them behind at a lowered speed and traverse a congestion area smoothly while mitigating shock waves.

Literature Review

In the past couple of decades, the practice of speed harmonization has been matured mainly through the VSL strategies which appeared to be more effective and efficient than other techniques such as VMS or the rolling speed harmonization *(9, 8)*. Up to date, advanced VSL strategy employs proactive approach that applies a control scheme beforehand by anticipating the complex behavior of dynamic systems *(10)*. Even though the proactive approach made the VSL systems more effective than ever before, they still remain sub-optimal since they use heuristic approach to search the best solution *(11)*. With an impetus that there was no speed harmonization algorithm which pursues optimal control yet, this study developed a control algorithm or tighten the inflow traffi-
cusing the Hamiltonian method through individual vehicular control. Given that the majority of

existing VSL strategies rely on macroscopic models which employ aggregated traffic data, considering microscopic behaviors of vehicles to design VSL strategy is expected to improve accuracy in representing traffic conditions, ability to reflect the occurrence of shock waves resulted from the behavior of individual drivers such as sudden deceleration, merging or lane changing *(10)*. Furthermore, to provide an environmentally-conscious strategy, our speed harmonization algorithm is designed to minimize acceleration variations which closely relate fuel consumption, while assuring an effective utilization of the roadway and safety elements via explicit constraints within the algorithm. In general, minimizing acceleration benefits the fuel consumption since internal combustion engines are optimized over steady state operating points (i.e., constant torque and speed) *(12)*. It is also proven by the fuel consumption model developed by Kamal et al. *(13)* which demonstrated a monotonic behavior of the fuel consumption with respect to the acceleration, and it becomes even more significant at higher vehicle speeds. The speed harmonization algorithm was evaluated using microscopic traffic simulations under the 100% automated vehicle environment, and the result was compared with one of the state-of-the-art speed harmonization algorithm as well as with no-control case.

Speed harmonization strategies can be broadly categorized in reactive and proactive approaches. The reactive approach control initiates the operation at a call upon a queue detected, and it uses immediate traffic condition information to determine a control strategy for the subsequent time interval. While the reactive strategy allows to remedy the bottleneck with real-time feedback forward operations, it has limitations related to time lag between the occurrence of congestion and a control implemented *(10)*. In contrast, the proactive approach has the capability of acting proactively, while anticipating the complex behavior of dynamic systems *(10)*. Thus, it can predict bottleneck formations before they even occur. Also, the nature of predictions of proactive VSL strategies allows for more systematic approach for network-wide coordination which supports system optimization, whereas reactive approach is restrained to a localized control logic.

Reactive Speed Harmonization

The first field implementation of speed harmonization is known as the VSL system in the German motorway A8 corridor in Munich stretched to the boundary of Salzburg, Austria in 1965 *(14)*. During the early 1960s, the US also implemented a VSL with Variable Message Sign (VMS) system on a portion of the New Jersey Turnpike *(9)*. These speed harmonization systems required human interventions to determine the messages or speed limits based on the conditions such as weather, traffic conditions and construction schedules. Since 1970s, advances in sensor technologies and traffic control systems allowed the speed harmonization automatically operated based on the traffic flow or weather conditions using various types of sensors. The earlier VMS and VSL implementations usually address safety issues under work zone areas or inclement weathers *(9)*. In 2007, the speed harmonization strategies aimed at improving traffic flow mobility. The VSL system in the M42 motorway at Birmingham, UK and Washington State Department of Transportation (WSDOT) *(15)*, the algorithms were automatically activated based on the pre-defined threshold of flow and speed collected from detectors embedded in the pavement and displayed the lowered speed limit within the control zone of pre-defined length *(16)*.

Development and evaluations of various VSL algorithms were actively practiced among academic researchers by using simulations tools. The evolution in VSL algorithms that can respond to current traffic more effectively allowed the performance of VSL to continuously enhance. Park and

Yadlapati *(17)* and Lin et al. (2004) developed reactive approach VSL algorithms to improve safety and mobility at work-zone areas. By determining the VSL control in responsive of the varying travel times in conjunction with the safety surrogate measures, the evaluations showed that the proposed VSL algorithms outperformed the existing VSL algorithms especially with the traffic demand fluctuations thanks to the responsive functions *(17)*. Juan et al. *(18)* conducted a simulation-based study and concluded that the performance of VSL can vary by the traffic volume levels. After reacing a particular traffic volume level, the benefit can become more apparent, or alternatively less obvious and therefore VSL needs to be intergrated with ramp metering control (18). Kwon et al. *(19)* developed a VSL algorithm which incorporated the function of identifying the moving jam based on the deceleration rate between adjacent spots. With the success in simulation-based evaluations, the VSL algorithm was implemented to the Twin Cities Metropolitan areas, MN. The field evaluation showed the reduction in average maximum deceleration by about 20% over the before case and improved the vehicle throughput at the bottleneck areas *(19)*.

Through their evolution, the reactive speed harmonization methods have consistently showed improvements in many aspects such as reliability, safety and environmental sustainability by providing adequate feedback to the dynamic traffic conditions. However, the capability of reactive control is limited as it only responds after a bottleneck occurs and heavily depend on the heuristic approach until the bottleneck is resolved.

Proactive Speed Harmonization

In order to achieve more systematical approach for preventing adverse impacts from impending shock waves, the proactive approach utilizing a prediction model was proposed. The concept of the proactive VSL strategy was first suggested by Alessandri et al. *(20)*. They adopted the Kalman Filter to estimate impending traffic status based on the time-series traffic measurements *(21)*. Given the estimated traffic flow, the VSL control algorithm produced control strategy to minimize various types of cost functions (e.g., average travel time, summation of square densities of all sections) using Powell's optimization method. Although this effort initiated the prediction-based VSL systems, the prediction using a time-series approach is not robust, especially when for unexpected traffic disturbances, since it is heavily relied on the empirical patterns.

A pioneering effort in developing a proactive VSL strategy was made by Hegyi et al. *(22)*. They proposed Model Predictive Control (MPC) for the proactive approach VSL systems *(22)*. MPC is a method for the dynamic traffic control problem that optimizes a cost function of the total time spent in the network by all drivers. It performs predictions by explicitly using macroscopic traffic models, and calculates the control scheme that minimizes an objective function. The key element of the Hegyi et al.'s algorithm *(22)* is that they focused on preventing traffic breakdown by decreasing the density of approaching traffic, rather than focusing on reducing the speed variances. Having the MPC framework which enabled a network-wide optimization, Hegyi et al. *(22)* coordinated a series of VSLs for the system optimization *(22)*. The idea behind the coordination of multiple VSLs was to resolve shock waves at the bottleneck as well as to prevent having upstream delays *(22)*.

Recognizing the capability of MPC method towards handling nonlinear and multi-variable models while providing the capability to consider a network-wide optimization, many researchers adopted MPC to develop proactive VSL methods. Lu et al. (2010) developed a MPC-based proactive VSL

algorithm with a different point of view; they focused on creating a discharge section immediate upstream of the bottleneck to regulate traffic flow into the bottleneck and remain close to its capacity. Carlson et al. *(23)* developed a proactive VSL algorithm aimed at improving solvability to improve solvability for large-scale network by adopting a discrete-time dynamic optimal control method using the suitable feasible-direction algorithm *(24)*. Furthermore, they applied a rolling horizon mode for providing efficient and simpler feedback control strategies, but their solution was sub-optimal *(23)*. The aforementioned MPC-based proactive VSL algorithms showed substantial improvements in vehicle throughput, safety, equity, and driver acceptance under microscopic simulation experiments *(23, 22, 25, 26)*. However, these methods impose some challenges in practical applications due to the computational complexity entailed with MPC. According to Frejo and Camacho *(11)*, the average computational time for MPC taken for a unit control horizon was about 316.47 seconds in a Pentium I3 with 3 GHz under the network used in their study. Also, the requirement of model development and calibration is another hurdle that weaken the robustness of their control algorithms.

Pursuing a practically applicable algorithm, Hegyi et al. (2008) developed a VSL algorithm so-called, SPECIALIST using shock wave theory. Based on the different traffic states along the freeway segments, their future traffic evolution pattern was predicted. By identifying the location of the front boundaries of shock waves and the active speed limits, VSL control scheme is determined in a way to maximize the discharge rate at the bottleneck (Hegyi et al., 2008). Their control logic is more robust in a way that the model only includes a few parameters that have physical interpretation, such as the maximum thresholds of speed and flow rate at which the traffic status is identified as having shock waves and the speed and the flow associated with free-flow traffic (Hegyi et al., 2008).

The performance of the speed harmonization has been varied by control strategies, characteristics of locality and driving behaviors. The travel time improvement has been a debatable point, i.e., there was no significant change, or even increase in travel time during peak hours *(19, 8)*. However, it has been widely agreed that the speed harmonization helps increase the vehicle throughput at the bottleneck: the vehicle throughput increased by 4-5% via VSL systems *(16)* and by 5-10 % via rolling speed harmonization implemented in European countries *(27)*. Speed harmonization also benefits the safety; personal injury crashes reduced about 30-35% in European experiments. Crash rate sustained even at the narrowed lanes during construction after the implementation of the VSL. The environmental impacts were substantial as well. Both the practice of VSL and rolling speed harmonization showed reduction in vehicle emissions by 4-10% depending on the pollutants *(27)*, and fuel consumption was reduced by 4% *(16)*.

Organization of the Paper

The structure of the remaining paper is as follows. In Section II we introduce the modeling framework, present the assumptions of our approach and formulate the problem. In Section III, we derive a closed-form analytical solution and show that the rear-end collision constraint does not become active. Finally, we provide simulation results in Section IV and concluding remarks in Section V.

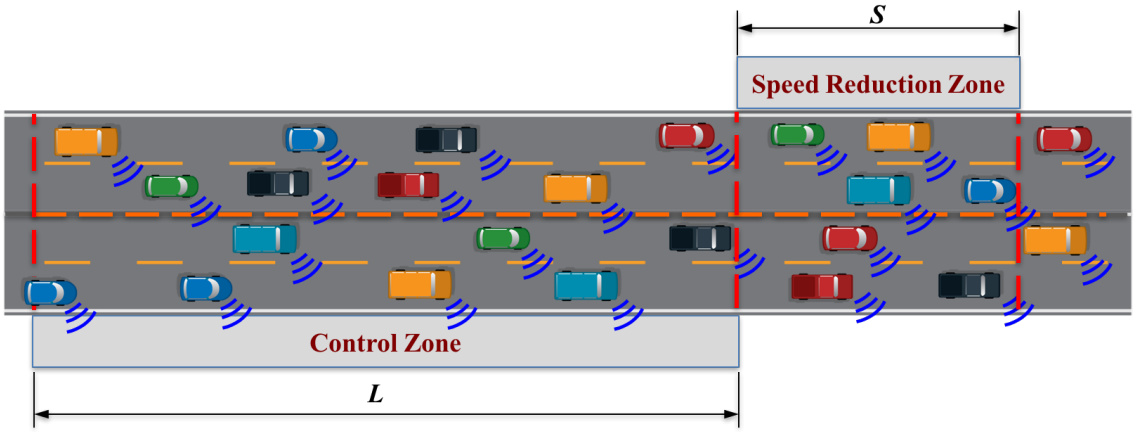

FIGURE 1 Speed harmonization at a speed reduction zone.

II. PROBLEM FORMULATION

We consider a highway with four lanes Figure 1 in each direction and a *speed reduction zone* with a length S. The highway has a *control zone*, and the distance from the entry of the control zone until the entry of the speed reduction zone is L.

Modeling Framework

We consider an increasing number of automated vehicles $N(t) \in N$ in each lane k, $k = 1, \ldots, 4$, where $t \in R^+$ is the time, entering the control zone. When a vehicle in a lane reaches the control zone at some instant t, we assign a unique identity $i = N(t) + 1$ which is an integer corresponding to the location of the vehicle in the queue for each lane k inside the control zone. The number $N(t)$ can be reset only if no vehicles are inside the control zone. To simplify notation, we restrict our attention to a single lane.

The proposed framework can be extended to multiple lanes, if each vehicle's identity include also the lane identity. If for example, there is a highway with m lanes, then we can assign an integer $i = N_k(t) + 1$, where N_k is the number of automated vehicles inside the control zone on the lane k, $k = 1, \ldots, m$.

Let $\mathcal{N}(t) = \{1, \ldots, N(t)\}$, be the queue in the lane associated with the control zone. We represent the dynamics of each vehicle $i \in \mathcal{N}(t)$, moving along a specified lane with a state equation

$$\dot{x}_i = f(t, x_i, u_i), \qquad x_i(t_i^0) = x_i^0, \tag{1}$$

where $t \in R^+$ is the time, $x_i(t)$, $u_i(t)$ are the state of the vehicle and control input, t_i^0 is the time that vehicle i enters the control zone, and x_i^0 is the value of the initial state. For simplicity, we assume that each vehicle is governed by a second order dynamics

$$\begin{aligned} \dot{p}_i &= v_i(t) \\ \dot{v}_i &= u_i(t) \end{aligned} \tag{2}$$

where $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, and $u_i(t) \in \mathcal{U}_i$ denote the position, speed and acceleration/deceleration (control input) of each vehicle i inside the control zone. Let $x_i(t) = \begin{bmatrix} p_i(t) & v_i(t) \end{bmatrix}^T$ denote

the state of each vehicle i, with initial value $x_i^0 = \begin{bmatrix} 0 & v_i^0 \end{bmatrix}^T$, taking values in the state space $\mathcal{X}_i = \mathcal{P}_i \times \mathcal{V}_i$. The sets $\mathcal{P}_i$, $\mathcal{V}_i$ and $\mathcal{U}_i$, $i \in \mathcal{N}(t)$, are complete and totally bounded subsets of R. The state space $\mathcal{X}_i$ for each vehicle i is closed with respect to the induced topology on $\mathcal{P}_i \times \mathcal{V}_i$ and thus, it is compact.

We need to ensure that for any initial state (t_i^0, x_i^0) and every admissible control $u(t)$, the system (1) has a unique solution $x(t)$ on some interval $[t_i^0, t_i^m]$, where t_i^0 is the time that vehicle $i \in \mathcal{N}(t)$ enters the control zone, and t_i^m is the time that vehicle i enters the speed reduction zone. The following observations from (2) satisfy some regularity conditions required both on f and admissible controls $u(t)$ to guarantee local existence and uniqueness of solutions for (2): a) the function f is continuous in u and continuously differentiable in the state x, b) the first derivative of f in x, f_x, is continuous in u, and c) the admissible control $u(t)$ is continuous with respect to t.

To ensure that the control input and vehicle speed are within a given admissible range, the following constraints are imposed.

$$\begin{aligned} u_{min} \leq u_i(t) \leq u_{max}, \quad \text{and} \\ 0 \leq v_{min} \leq v_i(t) \leq v_{max}, \quad \forall t \in [t_i^0, t_i^m], \end{aligned} \tag{3}$$

where u_{min}, u_{max} are the minimum deceleration and maximum acceleration respectively, and v_{min}, v_{max} are the minimum and maximum speed limits, respectively.

To ensure the absence of rear-end collision of two consecutive vehicles traveling on the same lane, the position of the preceding vehicle should be greater than, or equal to the position of the following vehicle plus a safe distance δ.

For each vehicle i, we define the control interval R_i as

$$\begin{aligned} R_i \Big\{ u_i(t) \in [u_{min}, u_{max}] \mid p_i(t) \leq p_k(t) - \delta, \\ v_i(t) \in [v_{min}, v_{max}], \forall i \in \mathcal{N}(t), |\mathcal{N}(t)| > 1, \forall t \in [t_i^0, t_i^f] \Big\}, \end{aligned} \tag{4}$$

where vehicle k is immediately ahead of i on the same road.

In the modeling framework described above, we impose the following assumptions:

Assumption 1: When the vehicles enter the control zone, the constraints are not active.

Assumption 2: For any vehicle $i - 1 \in \mathcal{N}(t)$ traveling on the same road and lane as vehicle $i \in \mathcal{N}(t)$, $v_{i-1}(t_i^0) \geq v_i(t_i^0) = v_i^0$.

Assumption 3: The speed for all vehicles inside the speed reduction zone is v_r, i.e., for all $i \in \mathcal{N}(t)$, $v_i(t_i^m) = v_i(t_i^f) = v_r$, where t_i^f is the time that each vehicle i exits the speed reduction zone.

Assumption 4: Each vehicle i has proximity sensors and can measure local information without errors or delays.

We briefly comment on the above assumptions. The first assumption assures that the solution will start from a feasible state and control input. The second assumption assures that the rear-end collision avoidance constraint does not become active at any time in (t_i^0, t_i^m). The feasibility enforcement analysis for the vehicles to satisfy such conditions imposed by Assumption 1 is discussed in *(28)*. The third assumption is a natural consequence of the speed reduction zone since

all vehicles should follow the speed designated in the zone. The fourth assumption might impose barriers in a potential deployment of the proposed framework. However, we could extend our results in the case that this assumption is relaxed, if the noise in the measurements and delays are bounded. In this case, we can determine the uncertainties of the state of the vehicle as a result of sensing and/or communication errors/delays, and account for these in the safety constraints.

Optimal Control Problem Formulation

We consider the problem of minimizing the congestion at the speed reduction zone, shown in Figure 1, with the optimal acceleration/deceleration of each vehicle in terms of fuel consumption under the hard safety constraints to avoid rear-end collision. The potential benefits of the solution of this problem are substantial. By controlling the vehicles in the upstream or tighten the inflow traffic, the speed of queue built-up decreases, and thus the congestion recovery time is also reduced. Even though the speed of each vehicle is reduced, the throughput of the highway is maximized.

When a vehicle enters the control zone, we assign a unique identity as described in the previous section. We formulate $N(t)$ decentralized problems that can be solved in real time. Before we proceed with the decentralized problem formulation we need to establish some definitions.

For each vehicle i when it enters a control zone, we define the *local observation set* $Y_i(t)$ as

$$Y_i(t)\Big\{p_i(t), v_i(t), t_1^m\Big\},\ \forall t \in [t_i^0, t_i^m], \tag{5}$$

where $p_i(t), v_i(t)$ are the position and speed of vehicle i inside the *control zone*, and t_1^m, is the time targeted for vehicle 1 in the FIFO queue to exit the speed reduction zone. Note that once the vehicle i enters the control zone, then immediately all information in $Y_i(t)$ becomes available to i.

We consider the problem of minimizing the control input at any time for each vehicle from the time t_i^0 it enters the control zone until the time t_i^m that enters the speed reduction zone under the hard safety constraints to avoid rear-end collision. The control problem of coordinating $N(t)$ vehicles in the lane can be formulated as

$$\min \frac{1}{2}\int_{t_i^0}^{t_i^m} u_i^2(t)\, dt, \tag{6}$$
$$\text{subject to : (2) and (3)},$$

with boundary conditions $p_i(t_i^0)$, $v_i(t_i^0)$, $p_i(t_i^m)$ and $v_i(t_i^m)$.

III. SOLUTION OF THE OPTIMAL CONTROL PROBLEM

For the analytical solution and real-time implementation of the control problem (6), we apply Hamiltonian analysis. In our analysis, we have assumed (Assumption 3.1) that when the vehicles enter the control zone, none of the constraints are active. However, this might not be in general true. For example, a vehicle may enter the control zone with speed higher than the speed limit. In this case, we need to solve an optimal control problem starting from an infeasible state. The feasibility enforcement analysis for the vehicles to satisfy such initial conditions is discussed in *(28)*.

297 **Analytical solution**

298 The solution of the problem including the rear-end collision avoidance constraint may become
299 intractable due to the numerous scenarios of activation/deactivation of the constraints. To address
300 this problem, the constrained and unconstrained arcs will be pieced together to satisfy the Euler-
301 Lagrange equations and necessary condition of optimality. Thus, it is not included in the analysis
302 below. However, we can guarantee rear-end collision avoidance at time t_i^m. In the following
303 section, we show that the rear-end collision avoidance constraint does not become active at any
304 time in (t_i^0, t_i^m) assuming it is not active at $t = t_i^0$.

From (6) and the state equations (2), the Hamiltonian function can be formulated for each vehicle $i \in \mathcal{N}(t)$ as follows

$$H_i\big(t, x(t), u(t)\big) = L_i\big(t, x(t), u(t)\big) + \lambda^T \cdot f_i\big(t, x(t), u(t)\big), \tag{7}$$

Thus

$$H_i\big(t, x(t), u(t)\big) = \frac{1}{2}u_i^2 + \lambda_i^p \cdot v_i + \lambda_i^v \cdot u_i, \tag{8}$$

305 where λ_i^p and λ_i^v are the co-state components. The necessary condition for optimality is

$$\frac{\partial H_i}{\partial u_i} = u_i + \lambda_i^v = 0, \tag{9}$$

306 From the last equation, the optimal control is given

$$u_i + \lambda_i^v = 0, \quad i \in \mathcal{N}(t). \tag{10}$$

307 The Euler-Lagrange equations yield

$$\dot{\lambda}_i^p = -\frac{\partial H_i}{\partial p_i} = 0 \tag{11}$$

308

$$\dot{\lambda}_i^v = -\frac{\partial H_i}{\partial v_i} = -\lambda_i^p. \tag{12}$$

309 From (11) we have $\lambda_i^p = a_i$ and (12) implies $\lambda_i^v = -(a_i t + b_i)$, where a_i and b_i are constants
310 of integration corresponding to each vehicle i. Consequently, the optimal control input (accelera-
311 tion/deceleration) as a function of time is given by

$$u_i^*(t) = a_i t + b_i. \tag{13}$$

312 Substituting the last equation into the vehicle dynamics equations (2) we can find the optimal speed
313 and position for each vehicle, namely

$$v_i^*(t) = \frac{1}{2}a_i t^2 + b_i t + c_i \tag{14}$$

314

$$p_i^*(t) = \frac{1}{6}a_i t^3 + \frac{1}{2}b_i t^2 + c_i t + d_i, \tag{15}$$

where c_i and d_i are constants of integration. These constants can be computed by using the initial and final conditions. Since we seek to derive the optimal control (13) online, we can designate initial values $p_i(t_i^0)$ and $v_i(t_i^0)$, and initial time, t_i^0, to be the current values of the states $p_i(t)$ and $v_i(t)$ and time t, where $t_i^0 \leq t \leq t_i^f$. Therefore the constants of integration will be functions of time and states, i.e., $a_i(t, p_i, v_i), b_i(t, p_i, v_i), c_i(t, p_i, v_i)$, and $d_i(t, p_i, v_i)$. To derive online the optimal control for each vehicle i, we need to update the integration constants at each time t. Equations (14) and (15), along with the initial and final conditions defined above, can be used to form a system of four equations of the form $\mathbf{T}_i\mathbf{b}_i = \mathbf{q}_i$, namely

$$\begin{bmatrix} \frac{1}{6}t^3 & \frac{1}{2}t^2 & t & 1 \\ \frac{1}{2}t^2 & t & 1 & 0 \\ \frac{1}{6}(t_i^f)^3 & \frac{1}{2}(t_i^f)^2 & t_i^f & 1 \\ \frac{1}{2}(t_i^f)^2 & t_i^f & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_i \\ b_i \\ c_i \\ d_i \end{bmatrix} = \begin{bmatrix} p_i(t) \\ v_i(t) \\ p_i(t_i^f) \\ v_i(t_i^f) \end{bmatrix}. \tag{16}$$

Hence we have

$$\mathbf{b}_i(t, p_i(t), v_i(t)) = (\mathbf{T}_i)^{-1}.\mathbf{q}_i(t, p_i(t), v_i(t)), \tag{17}$$

where $\mathbf{b}_i(t, p_i(t), v_i(t))$ contains the four integration constants $a_i(t, p_i, v_i)$, $b_i(t, p_i, v_i)$, $c_i(t, p_i, v_i)$, $d_i(t, p_i, v_i)$. Thus (13) can be written as

$$u_i^*(t, p_i(t), v_i(t)) = a_i(t, p_i(t), v_i(t))t + b_i(t, p_i(t), v_i(t)). \tag{18}$$

Since (16) can be computed online, the controller can yield the optimal control online for each vehicle i, with feedback indirectly provided through the re-calculation of the vector $\mathbf{b}_i(t, p_i(t), v_i(t))$ in (17). Similar results are obtained when the constraints become active as reported in *(29)*.

IV. SIMULATION FRAMEWORK AND RESULTS

To evaluate the effectiveness of the proposed optimal control algorithm, a simulation framework was established by integrating a controller and a simulator using the Visual C# programming environment. As presented in Figure 2, the optimal control algorithm described in the previous sections was coded using MATLAB language Dynamic Link Library (DLL) interface programming to allow data exchange with other external programs within the framework. A simulation test-bed network was developed under VISSIM, and it was integrated into the framework by using its COM interface.

The mobility measures such as travel time, average speed and vehicle throughput were directly obtained from VISSIM. Fuel consumption measure was estimated by using the polynomial metamodel proposed by Kamal et al. *(13)* which yielded vehicle fuel consumption as a function of speed, $v(t)$, and control input, $u(t)$ as in (19).

$$\dot{f}_v = \dot{f}_{cruise} + \dot{f}_{accel} \tag{19}$$

where $t \in R^+$ is the time, $\dot{f}_{cruise} = w_0 + w_1 \cdot v(t) + w_2 \cdot v^2(t) + w_3 \cdot v^3(t)$ estimates the fuel consumed by a vehicle traveling at a constant speed $v(t)$, and $\dot{f}_{accel} = u(t) \cdot (n_0 + n_1 \cdot v(t) + n_2 \cdot v(t)^2)$

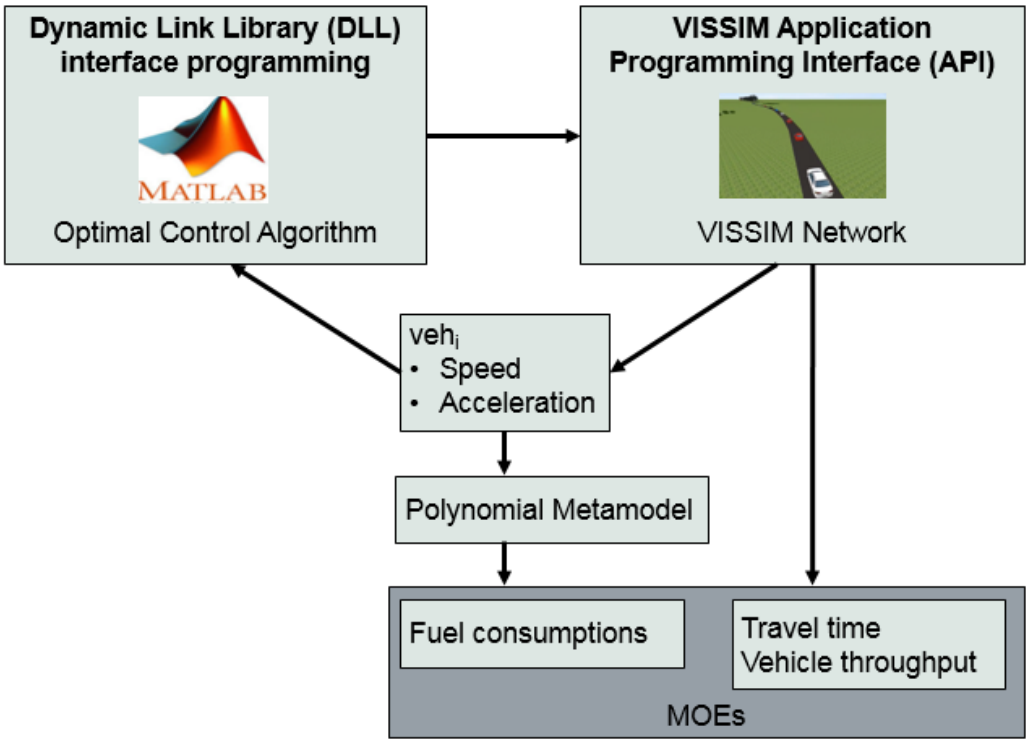

FIGURE 2 Overview of simulation framework

is the additional fuel consumption caused by acceleration $u(t)$. The polynomial coefficients w_n, $n = 0, \ldots, 3$ and r_m, $m = 0, 1, 2$ were calculated from experimental data. For the case studies we considered in this paper, all vehicles were the same with the parameters reported in *(13)*, where the vehicle mass was $M_v = 1,200\,kg$, the drag coefficient was $C_D = 0.32$, the air density was $\rho_a = 1.184\,km/m^2$, the frontal area was $A_F = 2.5\,m^2$, and the rolling resistance coefficient was $\mu = 0.015$.

Test-bed network

As noted, the proposed speed harmonization algorithm was implemented in the simulation test-bed network developed using the VISSIM microscopic traffic simulation program. A hypothetical test-bed network consists of about 2,000-meter single-lane corridor as shown in Figure 3. A 300-meter long speed reduction zone which is operated at the speed limit of 35 mph was located at the downstream of the network and a 300-meter long control zone was created immediate upstream the entrance of the speed reduction zone, so that the control algorithm effectively applies when speed deceleration is required.

The VISSIM model was carefully calibrated by referring to the guideline of the Highway Capacity Manual (HCM) 2010 *(30)*. The Chapter 15 of the HCM 2010 *(30)* presents that the capacity of two-lane highways under based conditions is 1,700 veh/hr, with a limit of 3,200 veh/hr for both directions. Without a possibility of having passing maneuvers from the opposite direction in the proposed test-bed network which is a one-way corridor, a maximum flow rate of 1,800 veh/hr was desired to achieve through calibration. To this end, the key parameters for the car-

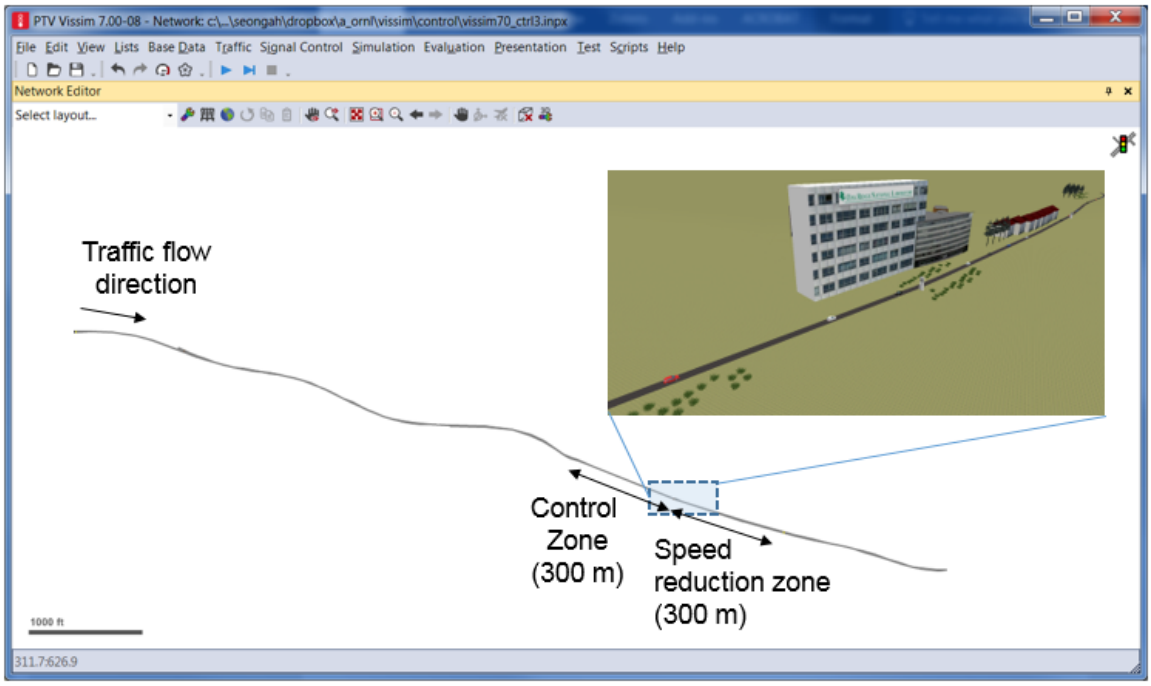

FIGURE 3 Test-bed network developed in VISSIM

following model which determine minimum distance between adjacent vehicles were assessed for calibration. In VISSIM, the minimum safety distance (dx_{safe}) which is defined as a distance a driver would maintain while following another vehicle can be expressed as shown in (20) *(31)*.

$$dx_{safe} = CC0 + CC1 \cdot v \tag{20}$$

where CC0 denotes a standstill distance between two vehicles (in feet), CC1 denotes a headway time (in seconds) which a driver wants to maintain at a certain speed and v represents average speed (ft/sec^2). With a good amount of calibration effort, the CC0 was used as the default value of 4.92 feet and the CC1 was adjusted to 1.2 seconds, thereby the maximum traffic flow was approximated about 1,800 veh/hr as desired.

Experimental set-up

To assess the impact of the optimal control algorithm under varying traffic volume conditions, three different volume cases were tested: (i) traffic volume of 1,620 veh/hr which is 10% less than the capacity (ii) traffic volume of 1,800 veh/hr at the capacity and (iii) traffic volume of 1,980 veh/hr which is 10% more than the capacity. For all scenario cases, the total simulation period was 1,000-second long comprising of 100-second warm-up period and 900-second of control algorithm implementation to avoid empty network situation during the algorithm applications. 5 replications of each simulation case were conducted to account for the effect of stochastic components of traffic and drivers' behaviors, and all produced statistically similar results with a 95% confidence level *(32)*.

The control parameters used for optimal control algorithm are summarized in Table 1. According

TABLE 1 Constraint Parameters of Optimal Control Algorithm

Parameter	Value
Min. speed	10 m/s
Max. speed	35m/s
Max. acceleration	4.5m/s^2
Max. deceleration	-4.5m/s^2
Min. gap distance	20 ft

to a guideline published by the Federal Highway Administration *(33)*, the maximum acceleration and deceleration are suggested as 10ft/sec^2(or 3.1m/s^2) and -15ft/sec^2(or -4.5m/s^2), respectively. Considering vehicle technical feasibility of automated vehicles, the maximum acceleration threshold was relaxed to 15ft/sec^2(or 4.5m/s^2) and the maximum deceleration was adopted as the guideline taking account for the safety and comfortable driving behaviors. The minimum gap distance of 20 ft was determined based on the shortest time headway of automated vehicles. According to Gouy et al. *(34)*, the minimum safety headway of automated vehicles was observed as 0.3 seconds under which a vehicle can travel about 20 ft with the maximum speed of 35 m/s^2 in this study. For a constancy between the controller and the traffic simulator, the optimal control strategy was calculated and updated every 0.1 seconds which is identical with the VISSIM microscopic simulator resolution.

To assess the performance of the optimal control algorithm, two comparison groups were developed: (i) a base case associated with human drivers based on the Wiedemann 99 psycho-physical car-following model and (ii) the state-of-the-art VSL algorithm called SPECIALIST (SPEed ControllIng ALgorIthm using Shock wave Theory) *(35)*. The SPECIALIST algorithm is a proactive VSL algorithm which projects a traffic conditions in the near future using the Model Predictive Control (MPC) method *(35)*. The highlight of the SPECIALIST is that the algorithm utilizes the shock wave theory to generate the control scheme (e.g., control speed and control duration), thus it does not require complicated computation and only has a few parameters with physical interpretations that helps for feasible field implementations *(35)*. In this study, the SPECIALIST algorithm was modeled using the C# programming and implemented in the VISSIM using its COM interface. Since the SPECIALIST algorithm bases on a mesoscopic model which utilizes the spot-based measurement collected at a fixed location and aggregated for a certain period of time, detector stations were evenly embedded at every 250 feet along the corridor to estimate the local traffic states. The traffic state of each detector station was estimated every 60 seconds by using the aggregated estimation of the latest 60-second interval, and the activation of VSL was examined every 60 seconds as well. Once the control scheme was generated, new measurement was not updated until the current control was finished. It is important to mention that the vehicles within the control zone were ensured to follow the VSL control scheme at 100% compliance rate without perception-reaction time. Such ideal condition was necessary for a fair comparison with the optimal control algorithm which assumed 100% automated vehicle environment. The SPECIALIST algorithm had several parameters that can be selected by the operator. For the best performance of the algorithm, the parameters were tuned with several iterations. The thresholds of v_{max} and q_{max} were chosen as 35 mph and 1,500 veh/hr, respectively, which were determined after empirical trials to find the minimum values above which traffic congestion was not observed under the VSL implemented at the 100% automated vehicle market penetrations. Remaining design parameters (e.g., the shock

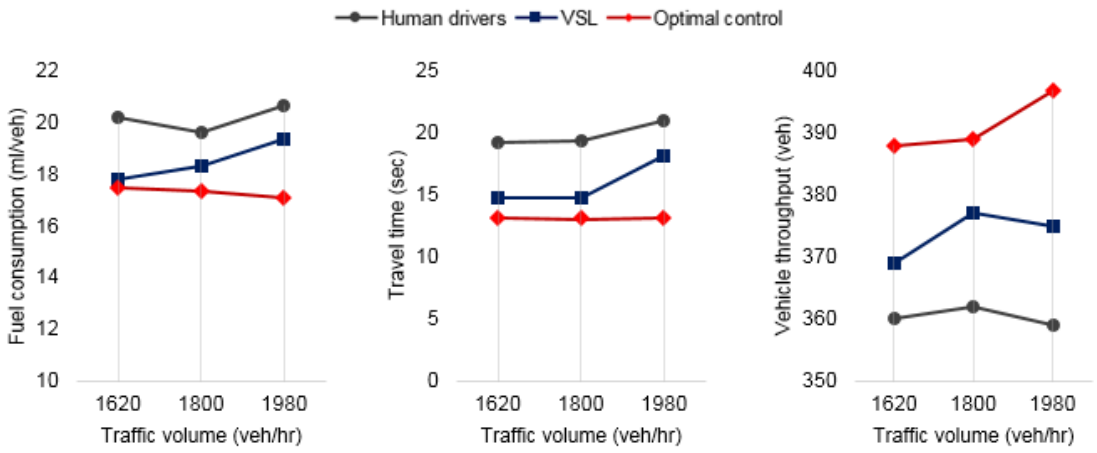

FIGURE 4 Comparisons among Base, VSL and Optimal control

wave propagation speed, the density within the zone where VSL is deployed, and the density and flow after the shock wave is resolved) were adopted from the earlier study *(35)*.

Results and analysis

Figure 4 presents mobility and fuel economy measures of no-control, the VSL algorithm and the proposed optimal control algorithm. When compared with no-control and VSL algorithm, the optimal control algorithm significantly reduced the per-vehicle fuel consumption by 12-17% over the no-control case and 2-12% over the VSL algorithm for the three traffic volume cases. Both the VSL algorithm and the optimal control algorithm improved the fuel consumption by ensuring vehicles to approach the speed reduction zone with less speed variation compared to the no-control case, but the VSL algorithm could not provide the optimized control scheme as the control scheme was heuristic solution. On the contrary, the optimal control algorithm provided vehicles in the control zone with the optimal strategy to approach the speed reduction zone, thereby the per-vehicle fuel consumption remain constant for all three traffic volume cases.

The optimal control algorithm also improved mobility. Travel time and vehicle throughput were improved for all three cases of traffic volumes over the no-control and the VSL algorithm. It is interesting to note that the VSL algorithm reduced the travel time when the traffic volume was less than or at the capacity, but it became less effective when the traffic volume exceeded the capacity. On the contrary, the optimal control reduced the travel time under the flow rate 10% more than the capacity, resulting in the travel time improvements of 32-28% over the base case and 11-28% over the VSL algorithm.

V. CONCLUDING REMARKS AND FUTURE RESEARCH

In this paper, we considered the problem of harmonizing in real time the speed of an increasing number of vehicles in a highway. We formulated the control problem and used Hamiltonian analysis to provide an analytical, closed-form solution that can be implemented in real time. The solution, when it exists, yields the optimal acceleration/deceleration of each vehicle to cross the speed reduction zone while maximizing the traffic throughput, and under the hard constraint of

collision avoidance. The effectiveness of the proposed solution is demonstrated through simulation and it is shown that the proposed approach can reduce significantly both fuel consumption and travel time.

In our proposed framework, we did not consider lane changing and we assumed that each vehicle can measure local information without errors or delays. The assumption of perfect information seems to impose barriers in a potential implementation and deployment of the proposed framework. Although it is relatively straightforward to extend our results in the case that this assumption is relaxed, future research should investigate the implications of having information with errors and/or delays to the system behavior. Finally, considering lane changing and mixed traffic, e.g., automated vehicles and human-driven vehicles, would eventually aim at addressing the remaining practical consequences of implementing this framework.

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REFERENCES

[1] Malikopoulos, A. A., A Duality Framework for Stochastic Optimal Control of Complex Systems. *IEEE Transactions on Automatic Control, Vol. 61, 10, pp. 2756-2765*, 2016 (forthcoming).

[2] Rios-Torres, J. and A. A. Malikopoulos, Automated and Cooperative Vehicle Merging at Highway On-Ramps. *IEEE Transactions on Intelligent Transportation Systems*, 2016 (forthcoming).

[3] Rios-Torres, J. and A. A. Malikopoulos, A Survey on the Coordination of Connected and Automated Vehicles at Intersections and Merging at Highway On-Ramps. *IEEE Transactions on Intelligent Transportation Systems*, 2016 (forthcoming).

[4] Malikopoulos, A. A. and J. P. Aguilar, An Optimization Framework for Driver Feedback Systems. *IEEE Transactions on Intelligent Transportation Systems*, Vol. 14, No. 2, 2013, pp. 955–964.

[5] Margiotta, R. and D. Snyder, *An agency guide on how to establish localized congestion mitigation programs*. U.S. Department of Transportation. Federal Highway Administration, 2011.

[6] Schrank, B., B. Eisele, T. Lomax, and J. Bak, *2015 Urban Mobility Scorecard*. Texas A& M Transportation Institute, 2015.

[7] INRIX Corporation, *INRIX 2015 Traffic Scorecard*, 2016.

[8] Roberts, M., *I-70 pace cars experiment is working, CDOT says - just not all the time*, 2012.

[9] Robinson, M., Examples of Variable Speed Limit Applications. Speed Management Workshop. Transportation Research Board., 2000.

[10] Khondaker, B. and L. Kattan, Variable speed limit: A microscopic analysis in a connected vehicle environment. *Transportation Research Part C: Emerging Technologies*, Vol. 58, 2015, pp. 146–159.

[11] Frejo, J. R. D. and E. F. Camacho, Global Versus Local MPC Algorithms in Freeway Traffic Control With Ramp Metering and Variable Speed Limits. *IEEE Transactions on Intelligent Transportation Systems*, Vol. 13, No. 4, 2012, pp. 1556–1565.

[12] Malikopoulos, A. A., P. Y. Papalambros, and D. N. Assanis, Online Identification and Stochastic Control for Autonomous Internal Combustion Engines. *Journal of Dynamic Systems, Measurement, and Control*, Vol. 132, No. 2, 2010, pp. 024504–024504.

[13] Kamal, M. A. S., M. Mukai, J. Murata, and T. Kawabe, Ecological Vehicle Control on Roads With Up-Down Slopes. *IEEE Transactions on Intelligent Transportation Systems*, Vol. 12, No. 3, 2011, pp. 783–794.

[14] Schick, P., Influence of Traffic Control Systems on Freeway Capacity and Stability of Traffic Flow. *University of Stuttgart*, 2003, p. 20.

[15] Brinckerhoff, P., T. Farradyne, and J. C. Burgess, *Active Traffic Management Concept of Operations*. Washington State Department of Transportation (WSDOT), 2008.

[16] Mott MacDonald Ltd., *ATM Monitoring and Evaluation 4-Lane Variable Mandatory Speed Limits 12 Month Report*. Highways Agency, Bristol UK, 2008.

[17] Park, B. and S. S. Yadlapati, Development and Testing of Variable Speed Limit Logics At Work Zones Using Simulation. 2003, Washington, DC., 2003.

[18] Juan, Z., X. Zhang, and H. Yao, Simulation Research and Implemented Effect Analysis of Variable Speed Limits on reeway, 2004, pp. 894–898.

[19] Kwon, E., C. Park, D. Lau, and B. Kary, Minnesota Variable Speed Limit System: Adaptive Mitigation of Shock Waves for Safety and Efficiency of Traffic Flows. Washington DC., 2011.

[20] Alessandri, A., A. Di Febbraro, A. Ferrara, and E. Punta, Optimal control of freeways via speed signalling and ramp metering. *Control Engineering Practice*, Vol. 6, No. 6, 1998, pp. 771–780.

[21] Welch, G. and G. Bishop, *An Introduction to Kalman Filter*. SIGGRAPH 2001, ACM, Inc., Department of Computer Sceience, University of North Carolina at Chapel Hill, 2001.

[22] Hegyi, A., B. Schutter, and J. Hellendoorn, Optimal coordination of variable speed limits to suppress shock waves. *Transportation Research Record: Journal of the Transportation Research Board*, , No. 1852, 2003, pp. 167–174.

[23] Carlson, R. C., I. Papamichail, M. Papageorgiou, and A. Messmer, Optimal mainstream traffic flow control of large-scale motorway networks. *Transportation Research Part C: Emerging Technologies*, Vol. 18, No. 2, 2010, pp. 193–212.

[24] Papageorgiou, M. and M. Marinaki, *A Feasible Direction Algorithm for the Numerical Solution of Optimal Control Problems*. Dynamic Systems and Simulation Laboratory, Technical University of Crete, Chania, Greece, 1995.

[25] Hegyi, A., B. De Schutter, and H. Hellendoorn, Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C: Emerging Technologies*, Vol. 13, No. 3, 2005, pp. 185–209.

[26] Hegyi, A., B. DeSchutter, and J. Hellendoorn, Optimal Coordination of Variable Speed Limits to Suppress Shock Waves. *IEEE Transactions on Intelligent Transportation Systems*, Vol. 6, No. 1, 2005, pp. 102–112.

[27] Fuhs, C., *Synthesis of Active Traffic Management Experience in Euripe and the United States*. US Department of Transportation Federal Highway Administration, 2010.

[28] Zhang, Y., C. G. Cassandras, and A. A. Malikopoulos, Optimal Control of Connected Automated Vehicles at Urban Traffic Intersections: A Feasibility Enforrcement Analysis. In *55th Conference on Decision and Control*, 2016 (in review).

[29] Malikopoulos, A. A., C. G. Cassandras, and Y. Zhang, Decentralized Optimal Control for Connected and Automated Vehicles at an Intersection, 2016, - arXiv:1602.03786.

[30] TRB, *Highway Capacity Manual 2010*. Transportation Research Board., Washington DC., 5th ed., 2010.

[31] PTV, *PTV VISSIM Traffic Simulation User Manual: Version 7*, 2014.

[32] VDOT, *Traffic Operations and Safety Analysis Manual (TOSAM) - Version 1.0*, 2015.

[33] Dowling, R., A. Skabardonis, and V. Alexiadis, *Traffic analysis toolbox volume III: guidelines for applying traffic microsimulation modeling software*, 2004.

[34] Gouy, M., C. Diels, A. Stevens, N. Reed, and G. Burnett, Do drivers reduce their headway to a lead vehicle because of the presence of platoons in traffic? A conformity study conducted within a simulator. *IET Intelligent Transport Systems*, Vol. 7, No. 2, 2013, pp. 230–235.

[35] Hegyi, A., S. P. Hoogendoorn, M. Schreuder, H. Stoelhorst, and F. Viti, SPECIALIST: A dynamic speed limit control algorithm based on shock wave theory. In *Intelligent Transportation Systems, 2008. ITSC 2008. 11th International IEEE Conference on*, IEEE, 2008, pp. 827–832.