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Connected and Automated Vehicle Merging at Highway On-Ramps

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Connected and Automated Vehicle Merging at Highway On-Ramps

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Abstract – Recognition of the necessity for connected and automated vehicles (CAVs) is gaining momentum. CAVs can improve both transportation network efficiency and safety through control algorithms that can harmonically use all existing information to coordinate the vehicles. This study addresses the problem of optimally coordinating CAVs at merging roadways to achieve a smooth traffic flow without stop-and-go driving. We present an optimization framework and an analytical, closed-form solution that allows online coordination of vehicles at the merging zone. The effectiveness of the efficiency of the proposed solution is validated through simulation and it is shown that coordination of vehicles can reduce fuel consumption and travel time significantly.

Keywords: Automated vehicles, merging highways, vehicle coordination, cooperative merging control, highway on-ramps, and cooperative driving

INTRODUCTION

The widespread use of the automobile is the source of traffic congestion and increasing traffic accidents. Although driver responses to various disturbances can cause congestion (*1*), intersections and merging roadways are the primary sources of bottlenecks (*2*). In 2014, congestion caused people in urban areas to spend 5.5 billion hours more on the road and to purchase an extra 2.9 billion gallons of fuel, resulting in a total cost estimated at $121 billion (*3*). Moreover, traffic congestion can produce driver discomfort, distraction, and frustration, which may encourage more aggressive driving behavior and further slow the process of recovering free traffic flow (*4*).

The increasing integration of energy, transportation, and cyber networks coupled with human interactions is giving rise to a new level of complexity in the transportation network. As we move to increasingly complex systems, new control approaches are needed to optimize the impact on system behavior of the interplay between vehicles at different traffic scenarios (*5–8*).

Connected and automated (CAVs) can provide shorter gaps between vehicles and faster responses while improving highway capacity. Several efforts reported in the literature have aimed at enhancing our understanding of the potential benefits of connected vehicle technologies. Li et al. (*9*) recently surveyed relevant research on improving transportation safety and efficiency using traffic lights and vehicle-to-infrastructure communication. There has been also a significant amount of work in developing approaches for improving both safety and traffic flow.

Ramp metering is a common method used to regulate the flow of vehicles merging into freeways to decrease traffic congestion (*10*). Although it has been shown that ramp metering can aim at improving the overall traffic flow and safety on freeways, there are several challenges associated with the interference between the traffic flows on adjacent roads. Different approaches to address these challenges, including the use of feedback control theory (*11*), (*12*), (*13*), (*14*), (*15*), optimal control (*16–18*) and heuristic algorithms (*19*, *20*), have been reported in the literature to date (*21*).

Given the recent technological developments, several research efforts have considered approaches to achieve safe and efficient coordination of merging maneuvers with the intention to avoid severe stop-and-go driving. Research efforts using either centralized or decentralized approaches have focused on coordinating CAVs in specific traffic scenarios, e.g., intersections, merging highways, etc (*22*). The overarching goal of such efforts is to yield a smooth traffic flow avoiding stop-and-go driving. In a centralized approach there is at least one task in the system that is globally decided for all vehicles by a single central controller. In decentralized approaches, the vehicles are treated as autonomous agents that attempt, through strategic interaction, to maximize their own efficiency. In this framework, each vehicle obtains information from other vehicles and roadside infrastructure to optimize specific performance criteria, e.g., efficiency, travel time, while satisfying the transportation system's physical constraints, e.g., stop signs, traffic signals.

One of the very early efforts in this direction was proposed in 1969 by Athans (*23*). Assuming a given merging sequence, Athans formulated the merging problem as a linear optimal regulator, proposed by Levine and Athans (*24*) to control a single string of vehicles, with the aim of minimizing the speed errors that will affect the desired headway between each consecutive pair of vehicles. Later, Schmidt and Posch (*25*) proposed a two-layer control scheme based on heuristic rules that were derived from observations of the non-linear system dynamics behavior. Similar to the approach proposed by Athans (*23*), Awal *et al*. (*26*) developed an algorithm that starts by computing the optimal merging sequence to achieve reduced merging times for a group of vehicles that are closer to the merging point.

Kachroo and Li (*27*) in 1997 used sliding mode control and designed longitudinal and lateral controllers to guide the vehicle until the merging maneuver is completed. The same year, Antoniotti (*28*, *29*) proposed a decentralized hybrid controller for keeping a safe headway between the vehicles in the merging process. In their work, there is no vehicle to vehicle communication but each vehicle decides the time to merge, yield, or exit the freeway based on the local information received from its own sensors. Ran et al. (*30*) used three levels of assistance for the merging process to select the available gap in the main road for the vehicle that is entering the merging ramp. Uno et al. (*31*) used the concept of virtual vehicle platooning for autonomous merging control. In this approach, a virtual vehicle is mapped onto the main road before the actual merging occurs. This concept was explored further by Lu and Hedrick (*32*) and Lu et al. (*33*), where a central controller identifies and interchanges relevant information with the vehicles that will be involved in the merging maneuver and each vehicle assumes its own control actions to satisfy the assigned time and reference speed.

Raravi et al. (*34*) proposed an approach in which, once a merging sequence has been defined, an optimization problem is solved to find the minimum time that each vehicle in the control area will take to reach the intersection. Milanes et al. (*35*) presented a fuzzy controller that uses the local information received to decide the accelerator and brake pedal position for each vehicle to achieve a smooth merging maneuver. The approach proposed by Marinescu et al. (*36*) builds upon the concept of slot-based traffic management, in which the intelligent vehicles drive inside a virtual slot. Ntousakis et al. (*37*) proposed two decentralized algorithms for automated merging control in which each vehicle uses information of the vehicles inside a cooperation area to determine the appropriate sequence to merge into the main road. Results showed that both algorithms performed safely and the traffic flow was kept at reasonable rates. More recently, the concept of cooperative merging, in which the vehicles on the main road adjust its speed to facilitate the merging process of the vehicle attempting to merge, was used in (*38*) and, a decentralized control framework, the analytical solution of which can coordinate CAVs in two adjacent intersections, was presented in (*39*).

Although previous research reported in the literature has aimed at enhancing our understanding of coordinating vehicles either at intersections, or merging roadways, deriving online an optimal closed-form solution for vehicle coordination in terms of fuel consumption still remains a challenging control problem. Depending on how they are formulated, the solutions based on optimization could impose a heavy computational load that will limit their potential for online implementation, which is the ultimate goal of any strategy.

This paper has two main objectives: 1) to formulate the problem of optimal vehicle coordination at merging roadways in terms of fuel consumption under the hard constraint of collision avoidance and 2) to derive online a closed-form solution in a centralized fashion. The research effort in this direction has been reported in (*40*), (*41*).

The contributions of this paper are 1) an analytical, closed-form solution using Hamiltonian analysis, and 2) the validation of the optimal solution through simulation and quantification of the implications for fuel consumption and travel time.

PROBLEM FORMULATION

Merging roadways are among the primary sources of bottlenecks generating traffic congestion resulting in severe stop-and-go driving and thus excessive fuel consumption. Figure 1 illustrates a common scenario in which a secondary one-lane road merges onto a main one-lane road. Typically, the vehicles on the secondary road have to yield to the vehicles on the main road and wait until the safest opportunity to merge onto the main road. On highly congested roads the merging process is even more tedious and undesirable stop-and-go traffic flow becomes unavoidable.

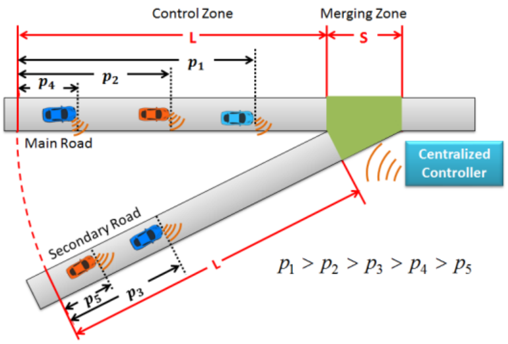


Figure 1. Merging Roads with connected and automated vehicles coordinated by a centralized controller

We consider the merging roadways of Figure 1. The region of potential lateral collision of the vehicles is called merging zone and has a length *S*. There is also a control zone and a centralized controller that can control the vehicles traveling inside the control zone. The distance from the entry of the control zone until the entry of the merging zone is *L*.

Modeling Framework

We consider an increasing number of automated vehicles $N(t) \in \mathbb{N}$, where $t \in \mathbb{R}$ is the time, entering the control zone (Figure 1). When a vehicle reaches the control zone at some instant t the controller assigns a unique identity $i = N(t) + 1$ that is an integer corresponding to the location of the CAV in a first-in-first-out (FIFO) queue for the control zone. If two, or more vehicles enter the control zone at the same time, then the controller selects randomly their position in the queue. The number $N(t)$ can be reset only if no vehicles are inside the control zone.

Let $\mathcal{N}(t) = \{1, \ldots, N(t)\}$ be the queue associated with the control zone. The dynamics of each vehicle i, $i \in \mathcal{N}(t)$, are represented with a state equation

$$\dot{x}_i = f(t, x_i, u_i), \quad x_i(t_i^0) = x_i^0, \tag{1}$$

where $t \in \mathbb{R}^+$ is the time, $x_i(t)$, $u_i(t)$ are the state of the vehicle and control input, t_i^0 is the time that vehicle i enters the control zone, and x_i^0 is the value of the initial state. For simplicity, we consider that each vehicle is governed by a second order dynamics

$$\begin{aligned} \dot{p}_i &= v_i(t) \\ \dot{v}_i &= u_i(t) \end{aligned} \tag{2}$$

where $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, and $u_i(t) \in \mathcal{U}_i$ denote the position, speed and acceleration/deceleration (control input) of each vehicle i. Let $x_i(t) = \left[p_i(t) \quad v_i(t) \right]^T$ denote the state of each vehicle i, with initial value $x_i^0(t) = \left[0 \quad v_i^0(t) \right]^T$, taking values in the state space $\mathcal{X}_i = \mathcal{P}_i \times \mathcal{V}_i$. The sets $\mathcal{P}_i$, $\mathcal{V}_i$ and $\mathcal{U}_i$, $i \in \mathcal{N}_i(t)$, are complete and totally bounded subsets of $\mathbb{R}$. The state space $\mathcal{X}_i$ for each vehicle i is closed with respect to the induced topology on $\mathcal{P}_i \times \mathcal{V}_i$ and thus, it is compact.

Optimization Problem Formulation

We seek to address the problem of coordinating online an increasing number of automated vehicles on two merging roadways. The objective is to derive an analytical solution that yields the optimal control input at any time in terms of fuel consumption. For the latter, we use the polynomial metamodel proposed in (*42*) yields vehicle fuel consumption as a function of the speed, v and control input, u.

To ensure that the control input and vehicle speed are within a given admissible range, the following constraints are imposed.

$$\begin{aligned} &u_{\min} \le u_i(t) \le u_{\max}, \text{ and} \\ &0 \le v_{\min} \le v_i(t) \le v_{\max}, \quad \forall t \in [t_i^0, t_i^f] \end{aligned} \tag{3}$$

where $u_{\min}$, $u_{\max}$ are the minimum deceleration and maximum acceleration respectively, and $v_{\min}$, $v_{\max}$ are the minimum and maximum speed limits respectively, t_i^0 is the time that vehicle i enters the control zone, and t_i^f is the time that vehicle i exits the merging zone.

To ensure the absence of rear-end collision of two consecutive vehicles traveling on the same lane, the position of the preceding vehicle should be greater than, or equal to the position of the following vehicle plus a predefined safe distance δ. Apparently, when there is only one vehicle in the control zone there is no concern of either rear-end collision, or lateral collision in the merging zone. Thus the following definition refer to the case when the queue $\mathcal{N}(t)$ contains more than one vehicle.

Definition 2.1: For each vehicle i, we define the control interval R_i as

$$\begin{aligned} R_i \triangleq \{ &u_i(t) \in [u_{\min}, u_{\max}] \,|\, p_i(t) \le p_k(t) - \delta, \\ &v_i(\text{t}) \in [v_{\min}, v_{\max}], \forall i \in \mathcal{N}(t), |\mathcal{N}(t)| > 1, \forall t \in [t_i^0, t_i^f] \}, \end{aligned} \tag{4}$$

where vehicle k is immediately ahead of i on the same road.

Definition 2.2: For each vehicle i, we define the set

Γ_i as the set of all positions along the lane where a lateral collision is possible, namely

$$\begin{aligned}\Gamma_i \triangleq \{p_i(t) \mid p_i(t) \in [L, L+S], \forall i \in \mathcal{N}(t),\\ \mathcal{N}(t) > 1, \forall t \in [t_i^0, t_i^f].\end{aligned} \tag{5}$$

To avoid lateral collision for any two vehicles i and j on different roads, the following constraint should hold

$$\Gamma_i \cap \Gamma_j = \varnothing, \forall t \in [t_i^0, t_i^f]. \tag{6}$$

The above constraint implies that only one vehicle, at a time, can be crossing the merging zone. If the length of the merging zone is long, then this constraint might not be realistic resulting in dissipating space and capacity of the road. However, the constraint is not restrictive in the problem formulation and it can be modified appropriately as described in the following section.

We impose the following assumption that is intended to enhance safety awareness.

Assumption 2.3: The vehicle speed inside the merging zone is constant.

We consider the problem of minimizing the control input at any time for each vehicle from the time t_i^0 it enters the control zone until the time t_i^m that enters the merging zone while reducing the gaps between the vehicles, under the hard safety constraints to avoid rear-end and lateral collision. The control problem of coordinating $N(t)$ vehicles can be formulated as

$$\min_{u_i \in R_i} \left(w_1 \frac{1}{2} \sum_{i=1}^{N(t)} \int_{t_i^0}^{t_i^f} u_i^2(t)dt + w_2 \sum_{i=2}^{N(t)} \left| t_i^m \left(u_{(1:i)}(t) \right) - t_{i-1}^m \left(u_{(1:i-1)}(t) \right) \right| \right) \tag{7}$$

$$\text{Subject to:} \quad \begin{aligned} &(2),\ \forall i \in \mathcal{N}(t) \\ &(6),\ \forall i \in \mathcal{N}(t), i \neq j, \end{aligned}$$

where w_1, w_2 are weighting factors that normalize the two terms in (7). Based on the Assumption (2.3), the time t_i^m that each vehicle i enters the merging zone is given by

$$t_i^m = t_i^f - \frac{S}{v_i(t_i^f)}, \tag{8}$$

where t_i^f is the time that each vehicle i exits the merging zone. The second term in (7) aims at minimizing the gaps between the vehicles, and thus fully exploiting the capacity of the road to avoid potential congestion. However, future research should investigate the existence of a potential trade-off between the two terms in (7).

Analytical Solution

When a vehicle enters a control zone, it receives a unique identity i from the centralized controller, as described in the previous section. Recall that $\mathcal{N}(t) = \{1, ..., N(t)\}$ is the FIFO queue of vehicles in the control zone. A vehicle index $i \in \mathcal{N}(t)$ also indicates which vehicle is closer to the merging zone, i.e., for any $i, k \in \mathcal{N}(t)$ with $i < k$ then $p_i < p_k$.

Definition 3.1: Each vehicle $i \in \mathcal{N}(t)$ belongs to at least one of the following two subsets: 1) $\mathcal{L}_i(t)$ contains all vehicles traveling on the same road with i, and 2) $\mathcal{C}_i(t)$ contains all vehicles traveling on different roads from i.

The time t_i^f that the vehicle i exits the merging zone is based on imposing constraints aimed at avoiding congestion in the sense of maintaining vehicle speeds above a certain value. There are two cases to consider:

1) If vehicle $i-1$ belongs to $\mathcal{L}_i(t)$, then to satisfy the second term of (7) both $i-1$ and i should have the *minimal safe distance* allowable, denoted by δ, by the time vehicle $i-1$ enters the merging zone, i.e.,

$$t_i^f = t_{i-1}^f + \frac{\delta}{v_i(t_i^f)}, \tag{9}$$

where $v_i(t_i^f) = v_i(t_i^0)$ as we designate the vehicles to exit the merging zone with the same speed they had when they entered the control zone. However, this is just a matter of specifying the final conditions of the vehicles when they exit the merging zone, and as such other alternatives could be considered depending on how we wish to formulate the problem.

2) If vehicle $i-1$ belongs to $\mathcal{C}_i(t)$, we constrain the merging zone to contain only one vehicle so as to avoid a lateral collision. Therefore, vehicle i is allowed to enter the merging zone only when vehicle $i-1$ exits the merging zone, where t_i^m is the time that the vehicle i enters the merging zone), i.e.,

$$t_i^f = t_{i-1}^f + \frac{S}{v_i(t_i^f)}, \tag{10}$$

where $v_i(t_i^f) = v_i(t_i^0)$. However, this constraint is not restrictive and we can easily modify it by relaxing (10) and either use only (9) for both cases, or use instead of S in (10) another desired value.

Note that this recursive relationship over vehicles in a control zone queue satisfies both the rear-end and lateral collision avoidance constraints. The rear-end collision avoidance constraint is satisfied at t_i^f through $t_i^f = t_{i-1}^f + \frac{\delta}{v_i(t_i^f)}$ and the lateral collision avoidance constraint through $t_i^f = t_{i-1}^f + \frac{S}{v_i(t_i^f)}$. The recursion is initialized whenever a vehicle enters a control zone, i.e., it is assigned $i=1$. In this case, t_1^f can be externally assigned as the desired exit time of this vehicle whose behavior is unconstrained except for (3). Thus the time t_i^f is fixed for each vehicle i. Consequently, instead of solving (7) for $w_2 \gg w_1$, we can solve an optimization problem for each vehicle in the queue separately

$$\min_{u_i} \frac{1}{2}\int_{t_i^0}^{t_i^f} u_i^2 \tag{11}$$

Subject to: (2), (4) $\forall i \in \mathcal{N}(t)$.

Hamiltonian Analysis

For the analytical solution and online implementation of the problem (11), we apply Hamiltonian analysis (*43*). To simplify the analysis we consider the unconstrained problem, and thus the optimal solution would not provide limits for the state and control. The constrained problem formulation is discussed in (*44*), and it would require the constrained and unconstrained arcs of the state and control input to be pieced together to satisfy the Euler-Lagrange equations and necessary condition of optimality (*45*). In the constrained problem formulation, we have to include two constraints for the speed and two constraints for the control (acceleration/braking) that would make the analysis intractable due to the numerous activation/deactivation scenarios of the constraints. So our approach yields the optimal solution as long as the control input and speed of each vehicle is within the imposed limits. Of course, the solution can be modified appropriately whenever the state/control constraints become active. However, the latter would result in a suboptimal solution.

From (11) and the state equations (2), the Hamiltonian function can be formulated for each vehicle $i \in \mathcal{N}(t)$ as follows

$$H_i(t,x(t),u(t)) = L_i(t,x(t),u(t)) + \lambda^T \cdot f_i(t,x(t),u(t)), \tag{12}$$

Thus

$$H_i(t,x(t),u(t)) = \frac{1}{2}u_i^2 + \lambda_i^p \cdot v_i + \lambda_i^v \cdot u_i, \tag{13}$$

where λ_i^p and λ_i^v are the co-state components.

The Hamiltonian allows finding the optimal control input, speed and position for each vehicle as a function of time, namely

$$u_i^*(t) = a_i t + b_i, \tag{14}$$

$$v_i^*(t) = \frac{1}{2}a_i t^2 + b_i t + c_i, \tag{15}$$

$$p_i^*(t) = \frac{1}{6}a_i t^3 + \frac{1}{2}b_i t^2 + c_i t + d_i, \tag{16}$$

where c_i and d_i are constants of integration. These constants can be computed by using the initial and final conditions. Since we seek to derive the optimal control (14) online, we can designate initial values $p_i(t_i^0)$ and $v_i(t_i^0)$, and initial time, t_i^0 to be the current values of the states $p_i(t)$ and $v_i(t)$ and time t, where $t_i^0 \le t \le t_i^f$. Therefore, the constants of integration will be functions of time and states, i.e., $a_i(t, p_i, v_i)$, $b_i(t, p_i, v_i)$, $c_i(t, p_i, v_i)$, and $d_i(t, p_i, v_i)$. To derive online the optimal control for each vehicle i, we need to update the integration constants at each time t. Equations (15) and (16), along with the initial and final conditions defined above, can be used to form a system of four equations of the form $\mathbf{T}_i\mathbf{b}_i = \mathbf{q}_i$, namely

$$\begin{bmatrix} \frac{1}{6}t^3 & \frac{1}{2}t^2 & t & 1 \\ \frac{1}{2}t^2 & t & 1 & 0 \\ \frac{1}{6}(t_i^f)^3 & \frac{1}{2}(t_i^f)^2 & t_i^f & 1 \\ \frac{1}{2}(t_i^f)^2 & t_i^f & t_i^f & 0 \end{bmatrix} \cdot \begin{bmatrix} a_i \\ b_i \\ c_i \\ d_i \end{bmatrix} = \begin{bmatrix} p_i(t) \\ v_i(t) \\ p_i(t_i^f) \\ d_i(t_i^f) \end{bmatrix}, \tag{17}$$

Hence we have

$$\mathbf{b}_i(t, p_i(t), v_i(t)) = \mathbf{T}_i^{-1} \cdot \mathbf{q}_i(t, p_i(t), v_i(t)), \tag{18}$$

where $\mathbf{b}_i(t, p_i(t), v_i(t))$ contains the four integration constants $a_i(t, p_i, v_i)$, $b_i(t, p_i, v_i)$, $c_i(t, p_i, v_i)$, $d_i(t, p_i, v_i)$. Thus (14) can be written as

$$u_i^*(t, p_i(t), v_i(t)) = a_i(t, p_i(t), v_i(t))t + b_i(t, p_i(t), v_i(t)). \tag{19}$$

Since (17) can be computed online, the controller can yield the optimal control online for each vehicle i, with feedback indirectly provided through the re-calculation of the vector $\mathbf{b}_i(t, p_i(t), v_i(t))$ in (18).

RESULTS

To validate the effectiveness of the efficiency of our analytical solution we simulated the merging scenario presented in previous sections in Matlab. In our simulation, the length of the control and merging zones is $L = 400\ m$ and $S = 30\ m$. We assume that each vehicle travels at a constant speed of 13.41 m/s before entering the control zone. When a vehicle reaches the control zone then the centralized controller designates its acceleration/deceleration until the vehicle exits the merging zone. All vehicles are assumed to have the characteristics described in Section II-B.

We considered four case studies: (1) coordination of 4 vehicles, 2 for each road, (2) coordination of 30 vehicles, 15 for each road, (3) coordination of 30 vehicles assuming the vehicles on the secondary road reach the control zone at a lower speed of 11.2 m/s, and (4) coordination of 30 vehicles that enter the control zone with 29 m/s. The solutions were compared to a baseline scenario where it was assumed that the vehicles on the main road have the right-of-way. That is, the vehicles on the secondary road have to come to a full stop before entering the merging zone. To quantify the benefits in fuel consumption, we used the polynomial metamodel in (*42*) as discussed in Section II-B.

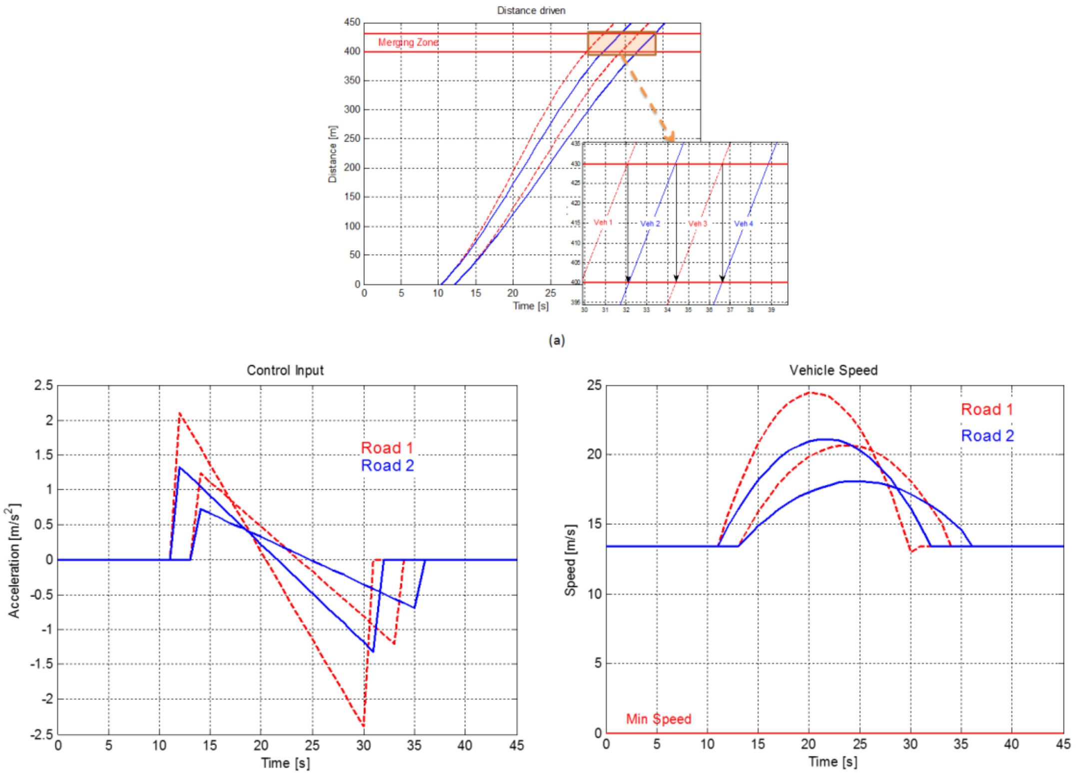
Case Study 1: Coordination of 4 vehicles

In this case study, we implemented the analytical solution for the coordination of 4 vehicles. The vehicles depart from the same position on each road. The purpose of this scenario is to validate that the controller will coordinate each vehicle to enter the merging zone only after the previous vehicle has already left (Figure 2a). Even though the vehicles start at the same initial positions on each road, the controller was able to derive online the optimal acceleration/deceleration by allowing only one vehicle at a time in the merging zone. The optimal acceleration/deceleration and speed profile for each vehicle are illustrated in Figure 2. Vehicle 1 accelerates first since it is cruising on the main road and has the right-of-way following by vehicle 2.

Case Study 2: Coordination of 30 vehicles

In this case study, the centralized controller coordinates 30 vehicles moving on two merging roads (15 vehicles on each road) with random initial positions and no limitations on the minimum or maximum speed, i.e., unconstrained problem. The controller is able to derive online the optimal control input for each vehicle by avoiding collision in the merging zone (Figure 3). We note that as the number of vehicles in the control zone on each road increases this has an impact on the acceleration/deceleration of each vehicle (Figure 3a). The controller accelerates the vehicles closer to the merging zone to create more space in the road for the following vehicles.

However, as the number of vehicles on the road increases and reaches its maximum capacity, eventually, the vehicles entering the control zone will need to decelerate, or even come to a full stop as imposed by the road capacity constraints. This is evident in Figure 3b, where the vehicles that are back in the queue need to decelerate as imposed by the safety constraints.

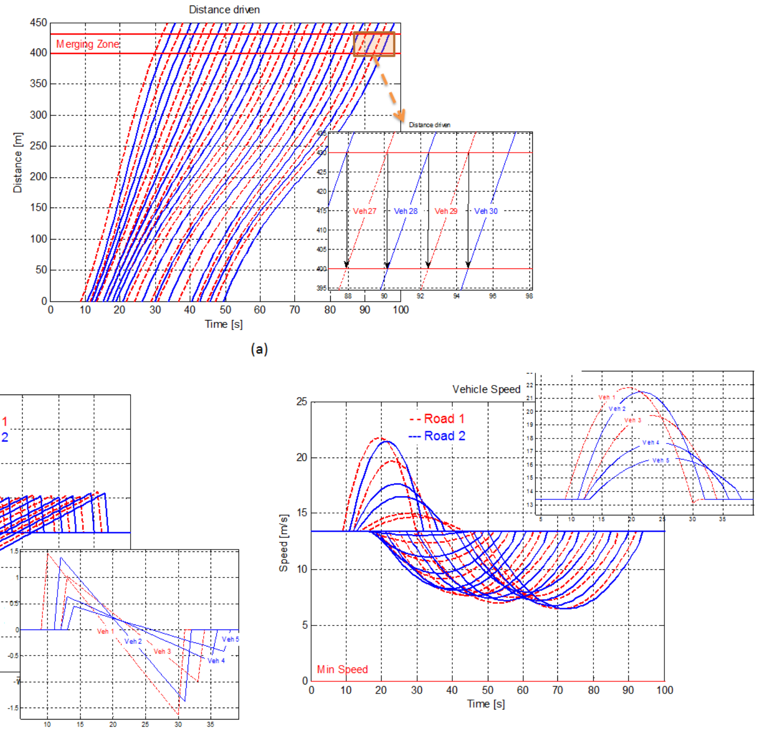


(a)

(b)

(c)

Figure 2. Position trajectories (a), control input (a) and speed profile (b) of the four vehicles for the case study 1



(a)

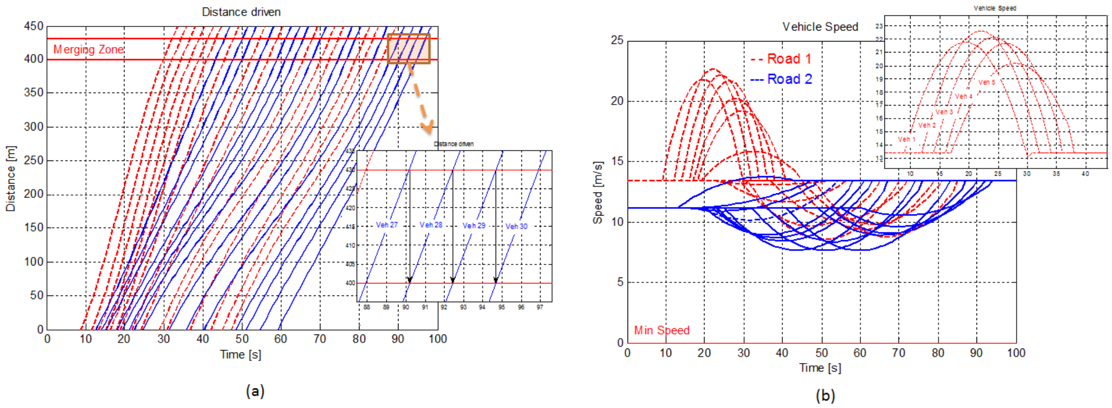
(b)

(c)

Figure 3. Position trajectories (a), control input (b) and speed profile (c) of the vehicles for the case study 2

Case study 3: Coordination with different initial speed for each road

In this case, we considered the coordination of 30 vehicles with different initial speeds for the main and secondary roads. The vehicles on the main road arrive at 30 mph and the vehicles on the secondary road will arrive at 25 mph. All the vehicles exit the merging zone at a desired speed of 30 mph. The position trajectory of the vehicles is illustrated in Figure 4a. The vehicles are able to merge without collision. Note also that the vehicles on the main road reach higher speed values (Figure 4b) than in the case study 2.



(a)

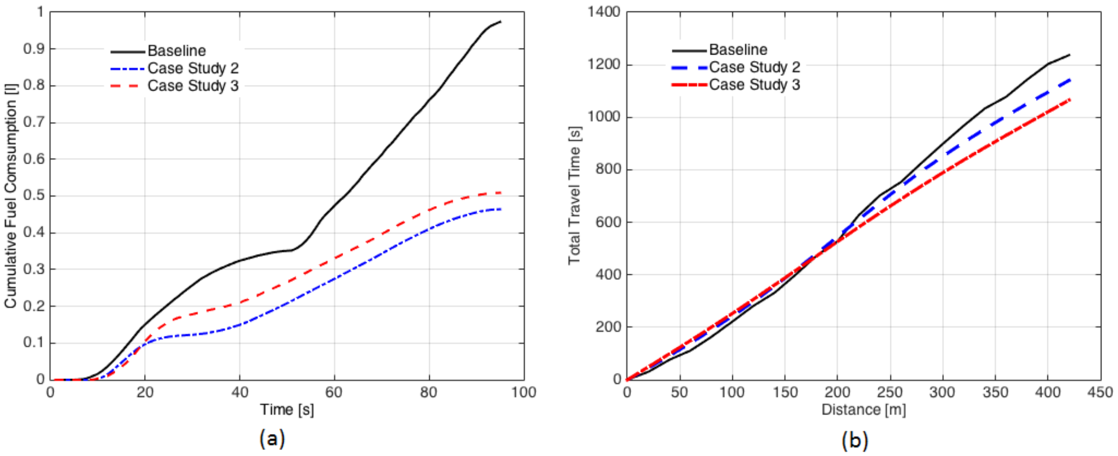
(b)

Figure 4. Position trajectories (a) and speed profile (b) of the vehicles in case study 3

Fuel consumption and travel time results

To compare fuel consumption benefits of vehicle coordination we considered a baseline scenario, in which the vehicles on the secondary road have to stop before the intersection to allow the vehicles in the main road to cross the merging zone. Only after all the vehicles on the main road have crossed, the vehicles on the secondary road start accelerating to reach again the maximum allowed speed.

The cumulative fuel consumption is higher in the baseline case compared to the case study 2 where the vehicles are coordinated through the centralized controller (Figure 5a). In particular, optimal vehicle coordination improves overall fuel consumption by 52.7% for the case study 2, and 48.1% for the case study 3 compared to the baseline scenario. The total travel time is also improved by 7.1%, and 13.5%, respectively (Figure 5b).



(a)

(b)

Figure 5. Cumulative fuel consumption (a) and travel time (b) for the baseline and case studies 2 and 3

Case study 4: Coordination at 65 MPH

Merging roadways are very common in highways. Thus we also considered a scenario where the vehicles enter the control zone at 29.05 m/s. The maximum and minimum speed limits inside the control zone were specified to be equal to 31.29 m/s and 22.35 m/s respectively.

In this case, however, the controller was unable to satisfy the safety constraints within the length of the control zone and the speed limits. To address this issue, we have two options: 1) increase the length of the control zone and 2) increase the speed limit. Since increasing the speed limit beyond 31.29 m/s might raise several safety concerns, we increased the length of the control zone to 1,200 m. However, we recognize that this might be unrealistically a long zone, and as such this fact indicates the potential limitations of the proposed approach. Nevertheless, the controller was able to coordinate the vehicles but some of the vehicles had to reach the speed limits, which indicates that eventually increasing also the speed limit might be inevitable.

CONCLUSIONS

In this paper, we addressed the problem of optimal coordination of CAVs at merging roadways. We formulated the problem as an unconstrained optimal control problem and we applied Hamiltonian analysis to derive an analytical, closed-form solution. The effectiveness of the efficiency of the proposed solution was validated through simulation and it was shown that vehicle coordination can reduce significantly both fuel consumption and travel time. The proposed approach allows the vehicles to merge without creating congestions and under the hard constraint of collision avoidance.

Ongoing research investigates the feasibility of the solution when at the time the vehicles enter the control zone some of the constraints are active and the computational implications. Future research should consider a more sophisticated transportation simulation model including more advanced vehicle models aimed at providing the practical implications of implementing such approach. Future research should also consider a diversity of vehicles and technologies, i.e., CAVs, non-CAVs, and also investigate the existence of a potential trade-off between fuel consumption and congestion.

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