Impact of Connected and Automated Vehicles on Traffic Flow

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Abstract—In this paper, we investigate the impact of connected and automated vehicles (CAVs) on traffic flow at merging roadways and develop a microscopic simulation framework to explore the implications on fuel consumption and travel time. In this framework, we use optimal control to simulate the behavior of CAVs and the Gipps car following model to capture the behavior of human-driven vehicles. The simulation results show that CAVs can contribute to significant fuel consumption and travel time reduction for diverse traffic conditions under average and high congestion scenarios. Furthermore, we show that CAVs allow for more stable traffic patterns even for high density traffic.

Keywords—Merging highways, connected and automated vehicles, energy implications, traffic analysis, cooperative merging control, car following, fundamental diagram.

I. INTRODUCTION

Merging roadways are among the primary sources of bottlenecks [1] due to the different maneuvers that drivers are required to coordinate in a limited period of time, to safely complete the merging process. The coordination of these maneuvers cause stop-and-go driving with significant implications in fuel consumption and traffic congestion [2], [3]. It has been shown that coordination of connected and automated vehicles (CAVs) can help addressing these issues by reducing human errors and providing shorter headway times, faster responses, and reduced travel time [4].

Several research efforts have considered approaches to achieve safe and efficient coordination of merging maneuvers with the intention to avoid severe stop-and-go driving. One of the very early efforts in this direction was proposed in 1969 by Athans [5]. Assuming a given merging sequence, Athans formulated the merging problem as a linear optimal regulator to control a single string of vehicles, with the aim of minimizing the speed errors that will affect the desired headway between each consecutive pair of vehicles. Later, Schmidt and Posch [6] proposed a two-layer control scheme based on heuristic rules derived from observations of the non-linear system dynamics behavior. Similar to the approach proposed in [5], Awal *et al.* [7] developed an algorithm that starts by computing the optimal merging sequence to achieve reduced merging times for a group of vehicles that are closer to the merging point.

More recently, the problem of coordinating vehicles that are wirelessly connected to each other at merging roadways was addressed in [8]. An analytical, closed-form solution was developed aimed at optimizing the acceleration of each vehicle online, in terms of fuel consumption, while avoiding collision with other vehicles at the merging zone. The framework was later extended to account a mixed traffic (CAVs interacting with human-driven vehicles) and analyze the impact of different penetration rates of CAVs on energy consumption [9].

There have also been some efforts towards enhancing our understanding of the effects of CAVs on traffic flow. A microscopic simulation model was presented in [10] to study the effects of an automated highway system on the average traffic speed. Li and Ioannou [11] developed a mesoscopic and a macroscopic traffic flow models based on the dynamics of intelligent cruise control vehicles to describe the traffic flow characteristics. The effectiveness of the models was demonstrated through simulations that revealed traffic flow differences with respect with the models representing manually driven vehicles. More recently, Talebpour and Mahmassani [12] presented a framework that uses different models and technology-related assumptions to simulate vehicles with district communication and level of automation capabilities.

While several studies have shown some benefits of CAVs in specific transportation scenarios, there are still open issues that need to be addressed. In particular, the prediction of the impact of CAVs on traffic flow, safety and fuel efficiency is one among many challenges that the community is currently facing.

In this paper, we make a preliminary effort to enhance our understanding of the impact that CAVs can pose on traffic flow by using a microscopic simulation framework. In particular, we study the impact of 100% penetration level of CAVs on traffic flow, considering a merging on-ramp as a transportation scenario. Furthermore, we investigate the related implications on fuel consumption and travel time under different traffic conditions. The remainder of the paper proceeds as follows. In section II, we present the modeling approach for the CAVs and the human-driven vehicles (HDVs). In Section III, we provide the simulation results and discussion. Finally, we draw concluding remarks in Section IV.

II. MODELING APPROACH

Our proposed approach is illustrated in Figure 1. To generate the data required for the analysis of the implications of CAVs in different traffic conditions, we create different traffic scenarios by assuming a set of average traffic flows and simulate each of them for two particular cases: 1) all the vehicles are CAVs using the optimal control framework proposed in [8] and 2) all the vehicles are human-driven. To model the behavior of human drivers, we adopt the Gipps car-following model [13]. Several studies have shown that this model can model driver behaviors with acceptable accuracy and it is used in traffic simulation software like AIMSUN [13], [14]. The details of these models are described in the following subsections.
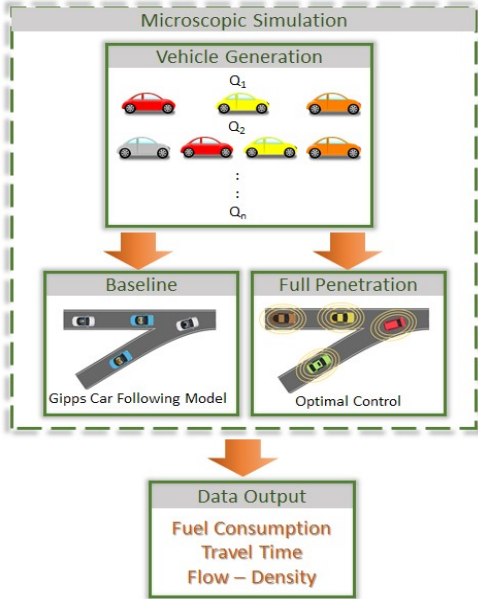

Figure 1. Approach to analyze the implications of connected and automated vehicles for different traffic conditions.

A. Vehicle Generation

To generate the different traffic scenarios, we use the shifted negative exponential distribution as proposed by the federal highway administration (FHWA) [15] aimed at deciding the inter-arrival time of the vehicles to the road section. According to this distribution, the vehicles will arrive at the entry node following a given average vehicular flow as defined in equations (1) and (2).

$$h = (H - h_{\min})[-\ln(1 - R)] + H - h_{\min}, \tag{1}$$

$$H = 3600 / Q_{avg}, \tag{2}$$

where h is the headway time [sec], H is a mean headway time [sec], $h_{\min}$ is a specified minimum headway [sec], R is a random number between 0 and 1, and Q_{avg} is the average vehicular flow [veh/sec].

We consider a merging on-ramp (Figure 2), where there is a control zone (pre-merging zone for human-driven vehicles) in which the CAVs will derive their optimal acceleration/deceleration in order to merge with an appropriate speed and headway with respect to the leading vehicle. There is also a merging zone in which the vehicles will complete the merging maneuver while keeping a safe distance from each other. The "control zone" has length L and the "merging zone" has length S.
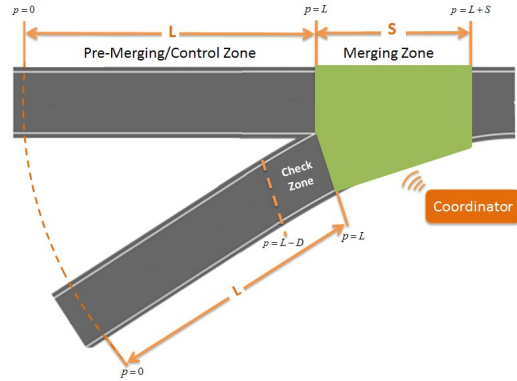

Figure 2. Merging roads scenario under investigation.

B. Modeling Framework for Connected and Automated Vehicles

We consider two single-lane merging roadways where the CAVs communicate with each other following the framework presented in [8]. According to this framework, once a vehicle enters the control zone, it shares information related to its speed and position. Then, based on a unique identity, each vehicle traveling inside the control zone uses this information available from all vehicles to compute its optimal acceleration/deceleration. The optimization problem and its solution is briefly discussed next, while more details can be found in [8].

We consider a number of automated vehicles $N(t) \in \mathbb{N}$, where $t \in \mathbb{R}$ is the time the vehicle enters the control zone. When a vehicle reaches the control zone at time t the controller assigns a unique identity $i = N(t) + 1$ that is an integer corresponding to the location of the CAV in a first-in-first-out (FIFO) queue for the control zone. If two or more vehicles enter the control zone at the same time, then the controller

selects randomly their position in the queue. We consider that each vehicle is governed by a second order dynamics

$$\dot{p}_i = v_i(t)$$
$$\dot{v}_i = u_i(t) \tag{3}$$

where $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, and $u_i(t) \in \mathcal{U}_i$ denote the position, speed and acceleration/deceleration (control input) of each vehicle. The sets $\mathcal{P}_i$, $\mathcal{V}_i$ and $\mathcal{U}_i$, $i \in \mathcal{N}_i(t)$, are complete and totally bounded subsets of the real numbers $\Re$.

1)   Optimization Problem Formulation
We seek to address the problem of coordinating online a number of CAVs on two merging roadways. The objective is to derive an analytical, closed form solution that yields the optimal control input at any time, in order to enter the merging zone with an appropriate speed and headway.

To ensure that the control input and vehicle speed are within a given admissible range, the following constraints are imposed.

$$u_{\min} \leq u_i(t) \leq u_{\max}, \text{ and}$$
$$0 \leq v_{\min} \leq v_i(t) \leq v_{\max}, \quad \forall t \in [t_i^0, t_i^f] \tag{4}$$

where $u_{\min}, u_{\max}$ are the minimum deceleration and maximum acceleration respectively, and $v_{\min}, v_{\max}$ are the minimum and maximum speed limits respectively, t_i^0 is the time that vehicle i enters the control zone, and t_i^f is the time that vehicle i exits the merging zone.

To ensure the absence of rear-end collision of two consecutive vehicles traveling on the same lane, the position of the preceding vehicle should be greater than, or equal to the position of the following vehicle plus a predefined safe distance δ. The following definition refer to the case when the queue $\mathcal{N}(t)$ contains more than one vehicle.

Definition 2.1: For each vehicle i, we define the control interval R_i as

$$R_i \triangleq \{u_i(t) \in [u_{\min}, u_{\max}] \mid p_i(t) \leq p_k(t) - \delta,$$
$$v_i(t) \in [v_{\min}, v_{\max}], \forall i \in \mathcal{N}(t), \mid \mathcal{N}(t) \mid > 1, \forall t \in [t_i^0, t_i^f]\}, \tag{5}$$

where vehicle k is immediately ahead of i on the same road.

Definition 2.2: For each vehicle i, we define the set Γ_i as the set of all positions along the lane where a lateral collision is possible, namely

$$\Gamma_i \triangleq \{p_i(t) \mid p_i(t) \in [L, L+S], \forall i \in \mathcal{N}(t),$$
$$\mathcal{N}(t) > 1, \forall t \in [t_i^0, t_i^f]. \tag{6}$$

To avoid lateral collision for any two vehicles i and j on different roads, the following constraint must hold

$$\Gamma_i \cap \Gamma_j = \varnothing, \forall t \in [t_i^0, t_i^f]. \tag{7}$$

The above constraint implies that only one vehicle, at a time, can be crossing the merging zone.

To derive an analytical solution of the optimal acceleration/deceleration, each CAV needs to know the time that it will be entering the merging zone. To accomplish this, we assign a coordinator to handle the information between the CAVs as follows. When a CAV reaches the control zone at some instant t, the coordinator assigns a unique identity i, which is an integer representing the location of the vehicle in the FIFO queue, $\mathcal{N}(t) = \{1, ..., N(t)\}$, inside the control zone. A vehicle index $i \in \mathcal{N}(t)$ also indicates which vehicle is closer to the merging zone, i.e., for any $i, k \in \mathcal{N}(t)$ with $i < k$ then $p_i < p_k$. Once a vehicle enters the control zone, it shares the time that will be exiting the merging zone, which is computed as explained below.

Definition 3.1: Each vehicle $j \in \mathcal{N}(t)$ belongs to at least one of the following two subsets: 1) $\mathcal{L}_i(t)$ contains all vehicles traveling on the same road with i, and 2) $\mathcal{C}_i(t)$ contains all vehicles traveling on different roads from i.

The time t_i^f that the vehicle i exits the merging zone is based on imposing constraints aimed at avoiding congestion in the sense of maintaining vehicle speeds above a certain value. There are two cases to consider:

a) If vehicle $i-1$ belongs to $\mathcal{L}_i(t)$, then both $i-1$ and i should have the *minimal safe distance* allowable, denoted by δ, by the time vehicle $i-1$ enters the merging zone, i.e.,

$$t_i^f = t_{i-1}^f + \frac{\delta}{v_i(t_i^f)}, \tag{8}$$

where $v_i(t_i^f) = v_i(t_i^0)$ as we designate the vehicles to exit the merging zone with the same speed they had when they entered the control zone.

b) If vehicle $i-1$ belongs to $\mathcal{C}_i(t)$, we constrain the merging zone to contain only one vehicle so as to avoid a lateral collision. Therefore, vehicle i is allowed to enter the merging zone only when vehicle $i-1$ exits the merging zone, where t_i^m is the time that the vehicle i enters the merging zone, i.e.,

$$t_i^f = t_{i-1}^f + \frac{S}{v_i(t_i^f)}, \tag{9}$$

where $v_i(t_i^f) = v_i(t_i^0)$. Note that this recursive relationship over vehicles in a control zone queue satisfies both the rear-end and lateral collision avoidance constraints. We can then solve an optimization problem for each vehicle in the queue separately

$$\min_{u_i} \frac{1}{2} \int_{t_i^0}^{t_i^f} u_i^2(t) dt \tag{10}$$

Subject to: (2), (4) $\forall i \in \mathcal{N}(t)$.

2) Hamiltonian Analysis

For the analytical solution and online implementation of the problem (10), we apply Hamiltonian analysis [16]. To simplify the analysis, we consider the unconstrained problem. Therefore, the optimal solution would not provide limits for the state and control [8]. The optimal closed form solution includes the optimal control input, speed and position for each vehicle as a function of time, namely

$$u_i^*(t) = a_i t + b_i, \tag{11}$$

$$v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i, \tag{12}$$

$$p_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i, \tag{13}$$

where a_i, b_i, c_i and d_i are constants of integration. To derive online the optimal control for each vehicle i, we need to update the integration constants at each sample time. Equations (12) and (13), along with the initial and final conditions on speed and position, are used to form a system of four equations of the form $\mathbf{T}_i \mathbf{b}_i = \mathbf{q}_i$, that can be solved to update the integration constants in real time.

C. Human-Driven Vehicles Model

We consider the merging roadways of Figure 2 and assume that all the vehicles behave according to the Gipps car-following model [17] which implies they do not receive information from nearby vehicles nor the infrastructure but use estimations of the behavior of their leading vehicle to decide a safe speed value at each sample time. As the vehicles get closer to the merging zone, we use a combination of Gipps car following model and heuristic control to represent the driver decisions for merging maneuvers.

The Gipps car following model adjust the driver behavior to keep a safe following distance from the leader vehicle or to travel at a desired speed in free traffic [14], [18], [19]. The speed v_f of the follower vehicle is computed as

$$v_f(t+\tau) = \min\{v_{f,acc}(t+\tau), v_{f,dec}(t+\tau)\}, \tag{14}$$

$$v_{f,acc}(t+\tau) = v_f(t) + 2.5 u_{f,max}\tau\left(1 - \frac{v_f(t)}{v_{f,max}}\right)\sqrt{0.025 + \frac{v_f(t)}{v_{f,max}}} \tag{15}$$

$$v_{f,dec}(t+\tau) = u_{f,min}\tau + \sqrt{u_{f,min}^2\tau^2 - u_{f,min}\left[\begin{array}{c}2(p_l(t) - p_f(t) - (L_{veh} + fd)) \\ -v_f(t)\tau - \frac{v_f(t)}{\hat{u}_{l,min}}\end{array}\right]}, \tag{16}$$

where the subscripts f, l identify the follower and the leader respectively, τ represents the "apparent" driver reaction time and corresponds to the simulation sample time, $v_{f,acc}$ is the speed when the vehicle is not constrained by the traffic, $v_{f,dec}$ is the speed when the vehicle is constrained by a leader in front, p is the vehicle position, v is the vehicle speed, $v_{f,max}$ is the maximum desired speed, $u_{f,max}$ is the maximum

desired acceleration, $u_{f,min}$ is the highest allowed braking value, $\hat{u}_{l,min}$ is the follower's estimation of the leader highest braking value, L_{veh} is the vehicle length and, fd is the desired headway distance when the vehicles are stopped. To ensure a collision-free trip, the follower's highest desired braking has to be greater or equal than the leader's highest braking value, namely

$$u_{f,min} \geq \hat{u}_{l,min}. \tag{17}$$

Each vehicle traveling on the main road will consider its preceding vehicle as its leader and will follow the speed dictated by the Gipps model until it reaches the merging zone, i.e., while its position $p_f(t)$ is less or equal than L. Once the vehicle is inside the merging zone, i.e., its position $p_f(t)$ is greater than L and less than $L+S$, it will evaluate whether another vehicle is merging from the secondary road. In such case, the follower traveling on the main road will start considering the merging vehicle as its new leader and the speed trajectory will be adjusted accordingly to avoid collision.

Similarly, each vehicle traveling on the secondary road will consider its preceding vehicle on the same road as its leader until it reaches a distance D from the merging zone, i.e., $p_f(t) = L - D$ (this section of the road of length D before the merging zone will be identified as the "check zone" for human-driven vehicles). Once the vehicle enters this zone, it checks for the closest gap to merge, i.e., it will identify a new potential leader (l) and a new potential follower ($f - 1$) in the main road. If the estimated time gap with the new potential follower is less than 2 *sec*, the vehicle will start decelerating to be able to stop before reaching the main road and to avoid lateral collision in the merging zone. The vehicle will then start evaluating the merging conditions again for the next available gap. Once a safe gap is identified, the vehicle will behave according to the Gipps car-following model, trying to avoid collisions with its new leader on the main road. The 2-sec threshold above is defined to follow the 2-sec rule, according to which, the time gap between two consecutive vehicles should be at least two seconds to allow a safe stop in case the vehicle in front has to suddenly brake.

III. SIMULATION RESULTS

To assess the impact of optimal coordination of CAVs for different traffic conditions, we simulated a merging on-ramp for two scenarios: 1) 0% CAVs penetration (baseline) and 2) 100% CAVs penetration. For each scenario, we generated a total of 19 different traffic conditions using (1) and (2) to compute the headway time for a total of 300 vehicles entering the control zone. We considered different Q_{avg} values between 300 veh/h and 1200 veh/h. We also considered that the control zone has a length $L = 400\ m$ and the merging zone a length $S = 30\ m$. Note that these values are not restrictive and they could be modified accordingly to represent specific

traffic segments. For the first scenario, we assumed that each driver attempts to reach and maintain a desired speed $v_{des} = 13.41$ m/s while keeping a safe distance from the leader vehicle. However, when a vehicle travels on the secondary road, we define a "check zone" of length γ to evaluate the merging conditions and decide whether to merge, or decelerate and wait for the next safe opportunity to merge. Similarly, in the scenario 2 we assumed that each CAV attempt to reach and maintain a desired speed $v_{des} = 13.41$ m/s and a safe distance from the leading vehicle before entering the control zone and after they leave the merging zone. Notably, the proposed approach is not restricted to the desired speed value used in this study, it can be modified according to the driver preferences. When a CAV reaches the control zone the controller designates its acceleration/deceleration until the vehicle exits the merging zone. To estimate the fuel consumption, we used the polynomial metamodel proposed in [20] that yields vehicle fuel consumption as a function of the speed and control input (acceleration/deceleration).

We simulated the baseline and the 100% CAVs penetration scenarios under each traffic flow condition and logged the time and speed for each vehicle entering the control zone. The logged data was aggregated every 30 sec to capture the macroscopic traffic flow and density for both scenarios.

The plot in Figure 3 shows that fuel consumption is reduced for all the simulated traffic conditions. For low traffic (300 to 600 veh/h) the fuel consumption reduction remains almost constant at almost 35%. The total fuel consumption varies significantly in the baseline case in medium and high traffic due to increased stop and go operation, reaching the maximum at a flow of 750 veh/h. The highest variation in the average traffic scenario is attributed to the fact that the vehicles still have some "freedom" to accelerate/decelerate as opposed to the case of high traffic where they are more "constrained" by the smaller headways and idling condition predominates.
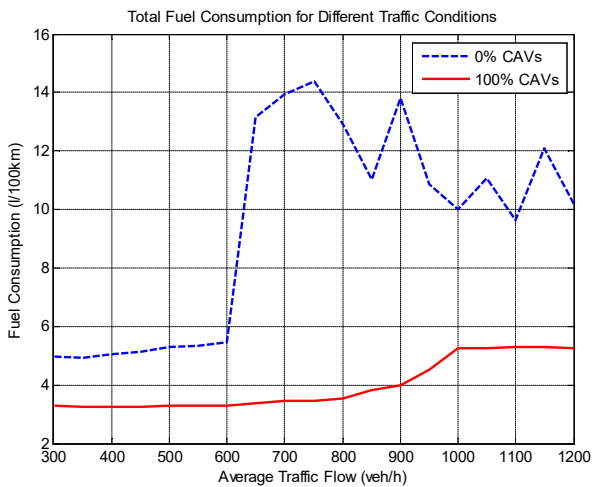

Figure 3. Fuel consumption for different traffic scenarios.

In contrast, for the 100% penetration scenario, the fuel consumption increases gradually for average traffic (from 600

veh/h to 1000 veh/h) but it reaches an almost constant value again for heavy traffic conditions. Note that for heavy traffic conditions, the percentage of fuel consumption reduction remains between 45% to 55%. The highest reduction percentage is reached at medium traffic conditions with almost 70%.

The total travel time (Figure 4) remains very close for both cases in low traffic conditions but can vary widely in the baseline case for medium and high traffic compared to the 100% CAVs penetration case, reaching the highest percentage of reduction in average traffic at 750 veh/h.
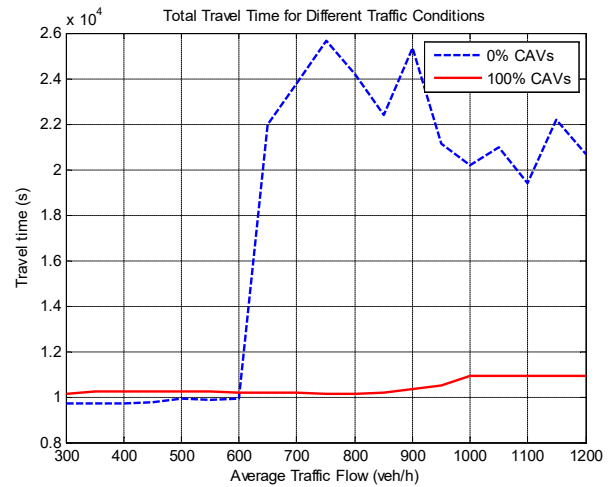

Figure 4. Travel time for different traffic scenarios.

To assess how the overall traffic is affected with full penetration of CAVs, we show the flow-density data in Figure 5.
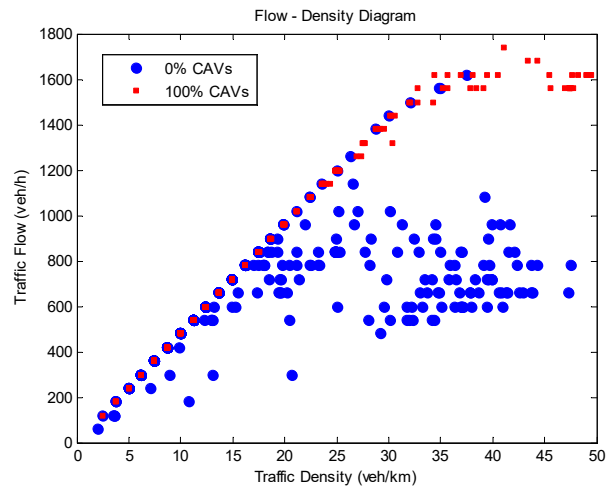

Figure 5. Flow-density data for all the simulated scenarios.

As expected, in the baseline case (blue dots) the points that represent the free traffic condition follow a linear pattern, while the data points representing congested traffic are more scattered. It is worthy to highlight that in the baseline case (0% penetration of CAVs) and for traffic densities between 26 veh/km and 40 veh/km, we have free traffic (represented by

the almost linear trend at the top of the plot) and congestion (represented by the scattered points in the plot) depending on the flow of vehicles that merge. Figure 5 shows that optimal coordination of CAVs can contribute with significant reduction of the wide variations in traffic flow and density that is generated by random traffic patterns. Small variations in traffic flow can still appear for higher traffic densities.

IV. Concluding Remarks

We are currently witnessing an increasing integration of energy and transportation, which, coupled with human interactions, is giving rise to a new level of complexity in the next generation transportation systems. The common thread that characterizes energy efficient mobility systems is their interconnectivity which enables the exchange of massive amounts of data; this, in turn, provides the opportunity for a novel computational framework to process this information and deliver real-time control actions that optimize energy consumption and associated benefits. CAVs provide the most intriguing and promising opportunity for enabling users (including individual vehicles and traffic control centers) to better monitor transportation network conditions and make better operating decisions to reduce energy consumption, greenhouse gas emissions, travel delays and improve safety.

The current amount of operational CAVs is not enough to capture significant data that can be used to assess the implications of CAVs on fuel consumption and traffic flow. In this paper, we made a preliminary effort to study the implications of optimal coordination of CAVs on merging roadways under different traffic conditions. We developed a simulation framework where the CAVs are optimally coordinated and the human-driven vehicles follow the Gipps car following model. The simulation results showed that fuel consumption can be significantly reduced under all traffic conditions, although the highest benefits are achieved at medium traffic flows. A similar trend was found for the travel time except that for low traffic conditions the results remain almost equal for both scenarios, i.e., baseline and full CAVs penetration. One important finding in our study is that full penetration of CAVs can contribute with more stable traffic patterns even when the traffic density is high. The analysis of the implications for different penetration levels of optimally coordinated CAVs is the subject of on-going research. Future work will also analyze the impacts of communication-related instabilities in the performance of the optimal coordination framework.

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