

An Optimal Coordination Framework for Connected and Automated Vehicles in two Interconnected Intersections

Behdad Chalaki, *Student Member, IEEE*, Andreas A. Malikopoulos, *Senior Member, IEEE*

Abstract—In this paper, we provide a decentralized optimal control framework for coordinating connected and automated vehicles (CAVs) in two interconnected intersections. We formulate a control problem and provide a solution that can be implemented in real time. The solution yields the optimal acceleration/deceleration of each CAV under the safety constraint at “conflict zones,” where there is a chance of potential collision. Our objective is to minimize travel time for each CAV. If no such solution exists, then each CAV solves an energy-optimal control problem. We evaluate the effectiveness of the efficiency of the proposed framework through simulation.

I. INTRODUCTION

Due to the increasing population and evolution of lifestyle, traffic congestion has become a significant concern in big metropolitan areas. By 2050, it is expected that 66% of the population will reside in urban areas. By 2030 there will be 41 Mega-cities (with more than 10M people) [1]. Schrank et al. [2] reported that in 2014 traffic congestion in urban areas in the US caused drivers to spend 6.9 billion additional hours on the road, burning 3.1 billion extra gallons of fuel.

One of the promising ways to address traffic congestion is to make cities integrated with information and communication technologies. Using CAVs is one of the intriguing ways towards making smart cities [3], [4]. As CAVs have gained momentum, different research efforts have been reported in the literature proposing coordination of CAVs. Most studies have focused on traffic bottlenecks such as merging roadways, urban intersections, and speed reduction zones [5]. More recently, a decentralized optimal control framework was established for coordinating online CAVs in different transportation scenarios, e.g., merging roadways, urban intersections, speed reduction zones, and roundabouts. The analytical solution using a double integrator model, without considering state and control constraints, was presented in [6], [7], and [8] for coordinating online CAVs at highway on-ramps, in [9] at roundabouts, and in [10], [11] at intersections. The solution of the unconstrained problem was also validated experimentally at the University of Delaware’s Scaled Smart City using 10 CAV robotic cars [12] in a merging roadway scenario and in a corridor [13]. The solution of the optimal control problem considering state and control constraints was presented in [14] at an urban intersection, without considering rear-end collision avoidance constraint

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Behdad Chalaki and Andreas A. Malikopoulos are with the Department of Mechanical Engineering, University of Delaware, Newark, DE 19716 USA (email: bchalaki@udel.edu; andreas@udel.edu).

though. The conditions under which the rear-end collision avoidance constraint never becomes active were discussed in [15]. The potential benefits of optimally coordinating CAVs in a corridor was presented in [16] while the implications of different penetration rates of CAVs can be found in [17].

Other efforts have used scheduling theory for addressing this problem [18], [18]–[21]. Colombo and Del Vecchio [19] proposed a least restrictive supervisor for intersection with collision avoidance constraints. Ahn [20] proposed the design of a supervisory controller by imposing a hard constraint on safety and studied its behavior in the presence of manual-driven cars. In a sequel paper [21], a supervisory controller was designed for an intersection with the ability to override human-driven control input in case of a future collision. Job-shop scheduling was used to solve this problem without considering rear-end collision avoidance constraint.

In this paper, we propose a decentralized optimal control framework of CAVs for two interconnected intersections using scheduling theory. We use a drone to act as a *coordinator* that can broadcast information with the CAVs. The proposed approach is different from other approaches in the literature in two aspects. First, most papers have considered single intersections, with only a few exceptions [10], [22], where the solution included two isolated coordinators and did not consider right and left turns. In our approach, we consider only one coordinator (the drone) and include right and left turns. Second, the majority of the papers in this area have used centralized scheduling which has some limitations in high traffic flow. In this paper, we use a decentralized scheduling approach that can make the system more robust in case one agent breaks down.

The remainder of the paper proceeds as follows. In Section II, we introduce the modeling framework, and the formulation of the minimization time-optimal and scheduling problems. In Section III, we present the analytical solution of the optimal control problem. Finally, we provide simulation results in Section IV, and concluding remarks in Section V.

II. PROBLEM FORMULATION

We consider two interconnected intersections shown in Fig. 1. A drone acting as a coordinator, stores information about the geometric parameters of the intersections, the paths of the CAVs crossing the intersections and schedules of CAVs. The intersection has a *control zone* (Fig. 1) inside of which the drone can communicate with the CAVs. The drone does not make any decision and only acts as a coordinator between CAVs. The area where lateral collision may occur is defined as a “*merging zone*.”

A. Modeling Framework

Let $N(t) \in \mathbb{N}$ be the number of CAVs inside the control zone at the time $t \in \mathbb{R}^+$ and $\mathcal{N}(t) = \{1, \dots, N(t)\}$ be a queue that designates the order that each CAV exiting the control zone. If two or more CAVs enter the control zone at the same time, the CAV with shorter path receives lower order in the queue; however, if the length of their path is equal, then their order is chosen arbitrarily.

We partition the roads around the intersections into 16 zones (Fig. 2) that belong to the set \mathcal{M} . The set \mathcal{M} has two subset, \mathcal{M}_1 and \mathcal{M}_2 . The subset \mathcal{M}_1 includes every zone except merging zones, and \mathcal{M}_2 includes the merging zones. Let \mathcal{I}_i denotes the path of each CAV. When CAV $i \in \mathcal{N}(t)$ enters the control zone, it creates a tuple of the zones $\mathcal{I}_i := [m_1, \dots, m_n]$, $m_n \in \mathcal{M}$, $n \leq 16$, that will cross.

Definition 1. For each CAV $i \in \mathcal{N}(t)$, we define a tuple $\mathcal{C}_{i,j}$ of conflict zones with CAV $j \in \mathcal{N}(t)$, $j < i$:

$$\mathcal{C}_{i,j} = [m \mid m \in \mathcal{I}_i \wedge m \in \mathcal{I}_j], \quad (1)$$

where the symbol “ \wedge ” corresponds to the logical “AND.”

In Definition 1, the condition $j < i$ implies that CAV i only considers CAVs that already within the control zone, i.e., their position in the queue is lower than i . Therefore, CAVs that are already inside the control zone do not need address potential conflicts with a new CAV that enters the control zone.

B. Vehicle model and assumptions

We represent the dynamics of CAV $i \in \mathcal{N}(t)$ with a state equation,

$$\dot{x}_i(t) = f(t, x_i(t), u_i(t)), \quad x_i(t_i^0) = x_i^0, \quad (2)$$

where $t \in \mathbb{R}^+$, and $x_i(t) = [p_i(t), v_i(t)]^T$, $u_i(t)$ are the state and control input of CAV i at time t . Let t_i^0 be the time that CAV $i \in \mathcal{N}(t)$ enters the control zone and $x_i^0 = [p_i^0, v_i^0]^T$ be

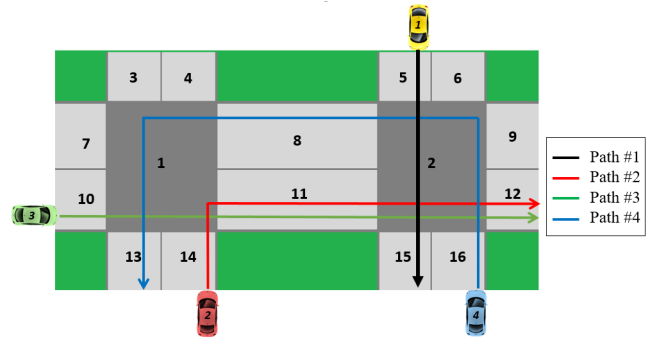


Fig. 2. Zones numbered topologically and the fixed path for each CAV is shown.

the state at this time. Let t_i^f be the time that CAV $i \in \mathcal{N}(t)$ exits the control zone.

We model each CAV $i \in \mathcal{N}(t)$ as a double integrator

$$\begin{aligned} \dot{p}_i &= v_i(t), \\ \dot{v}_i &= u_i(t), \end{aligned} \quad (3)$$

where $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, and $u_i(t) \in \mathcal{U}_i$ denote position, speed and acceleration. For each CAV $i \in \mathcal{N}(t)$ acceleration and speed is bounded with the following constraints:

$$u_{i,min} \leq u_i(t) \leq u_{i,max}, \quad (4)$$

$$0 < v_{i,min} \leq v_i(t) \leq v_{i,max}, \quad (5)$$

where $u_{i,min}$, $u_{i,max}$ are the minimum and maximum control inputs for each CAV $i \in \mathcal{N}(t)$ and v_{min} , v_{max} are the minimum and maximum speed limit respectively. For simplicity, we do not consider CAV diversity. Thus, in the rest of the paper, we set $u_{i,min} = u_{min}$ and $u_{i,max} = u_{max}$. The sets \mathcal{P}_i , \mathcal{V}_i and \mathcal{U}_i , $i \in \mathcal{N}(t)$, are complete and totally bounded subsets of \mathbb{R} .

Assumption 1. The path of each CAV is predefined and all CAVs in the control zone have access to this information through the drone.

Assumption 2. CAVs travel inside the merging zones with a constant speed which is known a priori.

Assumption 3. There is no error or delay in any communication between CAV to CAV, and CAV to drone.

The first assumption ensures that when a new CAV enters the control zone, it has access to the memory of the drone and discerns the path of CAVs that are inside the control zone. The second assumption is to enhance safety awareness inside the merging zones. Assumption 2 aim each CAV to solve a scheduling problem upon arriving the control zone. The third assumption may be strong but it is relatively straightforward to relax it, as long as the noise or delays are bounded.

Next, we describe three optimization problems: (1) a time-minimization problem, (2) a scheduling problem, and (3) an energy-minimization problem. Every time a CAV i enters the control zone, it solves a time minimization problem and a

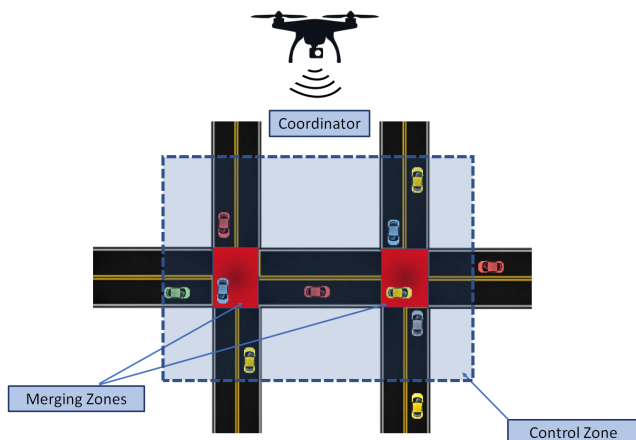


Fig. 1. Two interconnected intersections with a drone as a coordinator

scheduling problem. With time-minimization problem, each CAV seeks to minimize travel time in each zone. Then, the results of the time-minimization problem are used to solve the scheduling problem for each zone along their path. Since the scheduling problem considers safety, a solution may not exist. In this case, the CAV solves an energy-minimization problem to derive its optimal acceleration/deceleration from the time it enters until the time it exits the control zone.

C. Time Optimization Problem

We seek to develop a framework that minimizes the travel time of CAVs in the interconnected intersections using scheduling and optimal control theory. We formulate the following minimization problem:

Problem 1.

$$\begin{aligned} \min_{u_i \in \mathcal{U}_i} \quad & J_1(p_i(\cdot), v_i(\cdot), u_i(\cdot), t) = t_i^{e,m} - t_i^{s,m}, \\ \text{subject to:} \quad & (3), (4), (5), \end{aligned} \quad (6)$$

$$\text{and given } p_i^{s,m}, v_i^{s,m}, p_i^{e,m}, v_i^{e,m}.$$

In Problem 1, $[p_i^{s,m}, v_i^{s,m}]^T$ and $[p_i^{e,m}, v_i^{e,m}]^T$ are the initial and final states of CAV $i \in \mathcal{N}(t)$ at zone m , where $m \in \mathcal{I}_i \wedge m \in \mathcal{M}_1$. The time that CAV $i \in \mathcal{N}(t)$ exits the zone $m \in \mathcal{I}_i$ is denoted by $t_i^{e,m}$. For zone m , $m \in \mathcal{I}_i \wedge m \in \mathcal{M}_1$, the solution of Problem 1 yields the minimum time that CAV i can travel through zone m . It has been shown [23] that the optimal solution of (6) is bang-bang control, e.g., full-on forward ($u = u_{max}$) followed by full-on reverse ($u = u_{min}$). For the case that $m \in \mathcal{I}_i \wedge m \in \mathcal{M}_2$, since speed is constant, we do not need to minimize the time.

D. Scheduling Problem

Scheduling is a decision-making process which addresses the optimal allocation of resources to tasks over given time periods [24]. The problem of coordinating CAVs at the intersection is a typical scheduling problem [20], [21], [25]. Thus, in what follows we use scheduling theory to find the time that CAV $i \in \mathcal{N}(t)$ has to reach the zone $m \in \mathcal{I}_i$. Each zone $m \in \mathcal{M}$ represents a ‘‘resource,’’ and CAVs crossing this zone are the ‘‘jobs’’ assigned to the resource.

Definition 2. The time that a CAV $i \in \mathcal{N}(t)$ enters each zone $m \in \mathcal{I}_i$ is called ‘‘schedule’’ and is denoted by $T_i^m \in \mathbb{R}^+$. For CAV $i \in \mathcal{N}(t)$ we define a tuple of its schedules as follows:

$$\mathcal{T}_i = [T_i^m \mid m \in \mathcal{I}_i]. \quad (7)$$

For each zone $m \in \mathcal{I}_i$, $i \in \mathcal{N}(t)$, the schedule $T_i^m \in \mathbb{R}^+$ is bounded by

$$R_i^m \leq T_i^m \leq D_i^m, \quad (8)$$

where $R_i^m \in \mathbb{R}^+$ is the earliest feasible time that CAV $i \in \mathcal{N}(t)$ can reach the entry point of zone $m \in \mathcal{I}_i$, while $D_i^m \in \mathbb{R}^+$ is the latest feasible time. Moreover, $R_i^m \in \mathbb{R}^+$ and $D_i^m \in \mathbb{R}^+$ are called the ‘‘release time’’ and the ‘‘deadline’’ of the job respectively.

Definition 3. For each zone $m \in \mathcal{C}_{i,j}$, $i, j \in \mathcal{N}(t)$, $j < i$, the safety constraint can be restated as

$$|T_i^m - T_j^m| \geq h, \quad (9)$$

where $h \in \mathbb{R}^+$ is the safety time headway.

Problem 2. For each CAV $i \in \mathcal{N}(t)$ and each zone $m \in \mathcal{I}_i$ with safety time headway $h \in \mathbb{R}^+$ the scheduling problem is formulated as follows

$$\begin{aligned} \min_{T_i^m \in \mathbb{R}^+} \quad & J_2(T_i^m) = T_i^m, \\ \text{subject to:} \quad & (8), (9). \end{aligned} \quad (10)$$

Definition 4. The shortest feasible time that it takes for CAV $i \in \mathcal{N}(t)$ to travel through the zone $m \in \mathcal{I}_i$ is defined as the process time and is denoted by $P_i^m \in \mathbb{R}^+$.

The process time is the outcome of Problem 1, hence $P_i^m = t_i^{e,m} - t_i^{s,m}$.

Remark 1. In Problem 2, for each CAV $i \in \mathcal{N}(t)$ and $m \in \mathcal{I}_i$, the release time for zone $m+1 \in \mathcal{I}_i$ can be computed by the process time and schedule of zone $m \in \mathcal{I}_i$

$$R_i^{m+1} = P_i^m + T_i^m, \quad (11)$$

where $m+1 \in \mathcal{I}_i$ is the next zone that CAV $i \in \mathcal{N}(t)$ will cross after zone $m \in \mathcal{I}_i$.

Proposition 1. Let T_i^{m+1} be the entry time of CAV $i \in \mathcal{N}(t)$ at the zone $m+1 \in \mathcal{I}_i$ and R_i^{m+1} be the earliest time that CAV i can reach zone $m+1$. Then the feasible time-optimal solution of CAV i at the zone $m \in \mathcal{I}_i$ exist (it does not violate safety constraint at entry of zone $m+1$), if and only if $T_i^{m+1} = R_i^{m+1}$.

Proof. Due to space limitations the proof is omitted but can be found in [28]. ■

E. Energy Minimization Formulation

If at zone $m \in \mathcal{I}_i$, CAV $i \in \mathcal{N}(t)$ does not have a feasible time-optimal solution, then we impose i to solve an energy minimization problem. In this problem, the cost function is the L^2 -norm of the control input. The implications of the solution of the energy-optimal control is the minimization of transient engine operation, which has direct benefits both in fuel consumption and emissions [26].

Problem 3. The energy minimization problem is formulated as follows.

$$\begin{aligned} \min_{u_i \in \mathcal{U}_i} \quad & J_3(p_i(\cdot), v_i(\cdot), u_i(\cdot), T_i^m, T_i^{m'}) = \frac{1}{2} \int_{T_i^m}^{T_i^{m'}} u_i(t)^2 dt, \\ \text{subject to:} \quad & (3), (4), (5), \end{aligned}$$

$$\text{and given } p_i^{s,m}, v_i^{s,m}, T_i^m, p_i^{e,m}, v_i^{e,m}, T_i^{m'}, \quad (12)$$

where $[p_i^{s,m}, v_i^{s,m}]^T$ and $[p_i^{e,m}, v_i^{e,m}]^T$ are initial and final states of CAV $i \in \mathcal{N}(t)$ at zone $m \in \mathcal{I}_i$ respectively. The entry and exit time of CAV $i \in \mathcal{N}(t)$ from zone $m \in \mathcal{I}_i$ is denoted by T_i^m and $T_i^{m'}$ respectively.

Remark 2. The exit time of CAV $i \in \mathcal{N}(t)$ from zone $m \in \mathcal{I}_i$ denoted by $T_i^{m'}$ and is equal to the entry time to the zone $m+1 \in \mathcal{I}_i$ which is the next zone that CAV will cross after zone m .

$$T_i^{m'} = T_i^{m+1} \quad (13)$$

III. ANALYTICAL SOLUTION OF THE PROBLEMS

In this section we provide the analytical solutions of Problem 1 and Problem 3. CAVs solve the scheduling problem with mixed integer linear program. Since the closed form analytical solution of the time-minimization problem is available, the mixed integer linear program can be computed as CAVs enter the control zone. For the sake of simplicity in notation, in the following section we use $p_i^s, v_i^s, p_i^e, v_i^e$ instead of $p_i^{s,m}, v_i^{s,m}, p_i^{e,m}, v_i^{e,m}$.

A. Solution of the time minimization problem

The solution of Problem 1, using Hamiltonian analysis, includes solving a system of non-linear, non-smooth equations [23]. Pontryagin [27] solved the problem in a simpler way using a graphical approach. Since the problem's state space is two dimensional, Pontryagin noted that the optimal controls are either $u = u_{max}$, or $u = u_{min}$, which can be characterized as parabolas in two-dimensional state space. Based on the initial state, it should traverse on one of the parabolas to reach the intermediate point then traverse from the intermediate point to reach the target point. The aforementioned intermediate point is also called a switching point, since the control input will be changed at this point.

Proposition 2. Let v_i^e be the speed of CAV $i \in \mathcal{N}(t)$ at the end of the zone $m \in \mathcal{I}_i$. Let u_{max} and u_{min} be the maximum acceleration and deceleration respectively. If CAV i cruises from the initial state $x_i^s = [p_i^s, v_i^s]^T$ to the final position p_i^e , then the speed of CAV $i \in \mathcal{N}(t)$ at the end of the zone $m \in \mathcal{I}_i$ should be bounded as follows:

$$\sqrt{2u_{min} \cdot (p_i^e - p_i^s) + v_i^{s2}} \leq v_i^e \leq \sqrt{2u_{max} \cdot (p_i^e - p_i^s) + v_i^{s2}} \quad (14)$$

Proof. Due to space limitations the proof is omitted but can be found in [28]. ■

Lemma 1. If CAV $i \in \mathcal{N}(t)$ has a feasible time-optimal solution at zone $m \in \mathcal{I}_i$ (Proposition 1), it has at most one switching point.

Proof. Due to space limitations the proof is omitted but can be found in [28].

We have three cases to consider:

Case 1: In (14), if v_i^e is equal to the lower bound, CAV $i \in \mathcal{N}(t)$ arrives at the end of zone $m \in \mathcal{I}_i$ with maximum deceleration u_{min} . In this case, there is no switching point.

Case 2: In (14), if v_i^e is equal to the upper bound, CAV $i \in \mathcal{N}(t)$ arrives at the end of zone $m \in \mathcal{I}_i$ with maximum acceleration u_{max} . In this case, there is no switching point.

Case 3: In this case, from (14) the CAV has at least one switching point. Let $t_i^* \in \mathbb{R}^+$ be the minimum travel time.

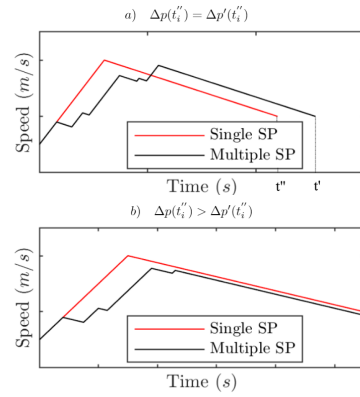


Fig. 3. Area under speed-time plot for multiple switching points and a single switching point.

Let $t_i' \in \mathbb{R}^+$ and $t_i'' \in \mathbb{R}^+$ be the time that it takes for CAV i to travel at zone m with more than one switching points, and exactly one switching point respectively. Let $\Delta p'(t)$ and $\Delta p(t)$ denote the distance traveled by CAV i at zone m at time t for more than one switching point and one switching point respectively, which can be represented by the area under $v-t$ diagram. First consider the case $\Delta p(t_i'') > \Delta p'(t_i'')$. $\Delta p(t_i'') = p_i^e - p_i^s$, hence $p_i^e - p_i^s > \Delta p'(t_i'')$, and since $p_i^e - p_i^s = \Delta p'(t_i')$, we have $\Delta p'(t_i') > \Delta p'(t_i'')$. Since $\Delta p'(t)$ is the distance travelled by CAV i , it is an increasing function. Therefore, $t_i' > t_i''$. Hence the minimum travel time is $t_i^* = t_i''$. For the case $\Delta p(t_i'') = \Delta p'(t_i'')$, it can be shown that for satisfying the endpoint speed condition, t' has to be greater than t'' . ■

An example of multiple switching point and a single switching point is shown in Fig. 3.

Theorem 1. Let $x_i^s = [p_i^s, v_i^s]^T$ and $x_i^e = [p_i^e, v_i^e]^T$ be the initial and final state of CAV $i \in \mathcal{N}(t)$. If CAV i has a feasible time-optimal solution at zone $m \in \mathcal{I}_i$ (Proposition 1), then it has to accelerate with $u = u_{max}$ and then decelerate with $u = u_{min}$.

Proof. Due to space limitations the proof is omitted but can be found in [28]. ■

Lemma 2. Let $x_i^0 = [p_i^s, v_i^s]^T$ and $x_i^e = [p_i^e, v_i^e]^T$ be the initial state and final state of CAV $i \in \mathcal{N}(t)$ travelling in zone $m \in \mathcal{I}_i$. Let $x_i^c = [p_i^c, v_i^c]^T$ be the intermediate state at switching point. Then, the intermediate state is

$$p_i^c = \frac{v_i^{e2} - v_i^{s2} + 2(u_{max}p_i^s - u_{min}p_i^e)}{2(u_{max} - u_{min})}, \quad (15)$$

$$v_i^c = \sqrt{v_i^{s2} + 2u_{max} \cdot (p_i^c - p_i^s)}. \quad (16)$$

Proof. Due to space limitations the proof is omitted but can be found in [28]. ■

Lemma 3. Let t_i^c , and t_i^e be the time at intermediate and final state for CAV $i \in \mathcal{N}(t)$ at zone $m \in \mathcal{I}_i$ respectively. Then t_i^c and t_i^e are

$$t_i^c = \frac{v_i^c - v_i^s}{u_{max}}, \quad (17)$$

$$t_i^e = \frac{v_i^c - v_i^s}{u_{max}} + \frac{v_i^e - v_i^c}{u_{min}}. \quad (18)$$

Proof. Due to space limitations the proof is omitted but can be found in [28]. ■

Theorem 2. For CAV $i \in \mathcal{N}(t)$ at zone $m \in \mathcal{I}_i$, the process time P_i^m is

$$\begin{cases} P_i^m = t_i^e & m \in \mathcal{M}_1 \\ P_i^m = \frac{p_i^e - p_i^s}{v_z} & m \in \mathcal{M}_2 \end{cases}, \quad (19)$$

where v_z is the constant speed inside the merging zones $m \in \mathcal{M}_2$.

Proof. Due to space limitations the proof is omitted but can be found in [28]. ■

Remark 3. If CAV $i \in \mathcal{N}(t)$ has a feasible time-optimal solution at zone $m \in \mathcal{I}_i$ (Proposition 1), the real time feedback control is

$$u_i(t) = \begin{cases} u_{max} & \text{if } T_i^m \leq t < T_i^m + t_i^c \\ u_{min} & \text{if } T_i^m + t_i^c \leq t \leq T_i^m + t_i^e \end{cases}. \quad (20)$$

Substituting (20) in (3), we can find the optimal position and speed for each CAV $i \in \mathcal{N}(t)$ at zone $m \in \mathcal{I}_i$, namely

$$v_i^*(t) = u_i t + b_i, \quad (21)$$

$$p_i^*(t) = \frac{1}{2} u_i t^2 + b_i t + c_i, \quad (22)$$

where b_i, c_i are integration constants, which can be computed by using initial and final conditions in (6). In particular, for $t \in [T_i^m, T_i^m + t_i^c)$, using (21) with the initial condition $v_i(T_i^m) = v_i^s$, (22) with the initial condition $p_i(T_i^m) = p_i^s$, a system of equations can be formulated in the form of $\mathbf{E}_i \mathbf{b}_i = \mathbf{q}_i$:

$$\begin{bmatrix} T_i^m & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} b_i \\ c_i \end{bmatrix} = \begin{bmatrix} p_i^s - \frac{1}{2} u_{max} \cdot (T_i^m)^2 \\ v_i^s - u_{max} \cdot T_i^m \end{bmatrix}. \quad (23)$$

However, for $t \in [T_i^m + t_i^c, T_i^m + t_i^e]$, using (21) with the final condition $v_i(T_i^m + t_i^e) = v_i^e$, (22) with the final condition $p_i(T_i^m + t_i^e) = p_i^e$, a system of equations will be as following:

$$\begin{bmatrix} T_i^m + t_i^e & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} b_i \\ c_i \end{bmatrix} = \begin{bmatrix} p_i^e - \frac{1}{2} u_{min} \cdot (T_i^m + t_i^e)^2 \\ v_i^e - u_{min} \cdot (T_i^m + t_i^e) \end{bmatrix}. \quad (24)$$

B. Solution of the Energy minimization problem

The derivation solution of the analytical, closed-form solution of Problem 3 has been presented [14], [15]. If none of the constraints are active the optimal control input, speed, and position of each CAV $i \in \mathcal{N}(t)$ are $u_i^* = a_i t + b_i$, $v_i^* = \frac{1}{2} a_i t^2 + b_i t + c_i$, and $p_i^* = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i$. In above equations a_i, b_i, c_i, d_i are integration constants, which can be found by plugging initial state at zone $m \in \mathcal{I}_i$ $p_i(T_i^m) = p_i^s, v_i(T_i^m) = v_i^s$ and final states $p_i(T_i^{m'}) = p_i^e, v_i(T_i^{m'}) = v_i^e$ in equations. Thus, a system of equations can be formed in form of $\mathbf{T}_i \mathbf{b}_i = \mathbf{q}_i$.

IV. SIMULATION

To evaluate the effectiveness of the proposed framework, we considered the coordination of 16 CAVs in simulation. We considered two symmetrical intersections, where the length of each road connecting to the intersections is 400 m, and the length of the merging zones are 30 m. The minimum time headway is 1.5 s. The maximum acceleration limit is set to 3 m/s² and the minimum deceleration limit is -3 m/s². The speed at merging zones is set 15 m/s. The CAVs enter the control zones from four different paths in order (shown in Fig. 2) at random time with uniform distribution between 0 and 20 s. Initial time were checked to not violate the safety constrained (9) at entrance.

The speed profile for the first 4 CAVs is shown in Fig. 4. CAVs 1–4 enter the control zone at random time, from path 1 to path 4 respectively. It can be seen in Fig. 4 that CAV 1, CAV 2 and CAV 4 are following time-optimal trajectory on each zone of their path. They accelerate in each zone $m \in \mathcal{M}_1$ until certain time (switching point) and then decelerate to reach the next zone. However, CAV 3 has a quadratic form at the beginning of its path. The quadratic form of velocity profile results from solving the energy-minimization problem implying that the time-optimal solution was not feasible for its first zone based on the schedule tuple (see the Table I). Therefore, acceleration/deceleration is found through solving an energy-optimal control problem. Moreover, the relative position of CAV 2 and CAV 3 with the conflict tuple $C_{3,2} = [1, 11, 2, 12]$ is shown in Fig. 5 for zones which rear-end collision may occur.

TABLE I

SCHEDULES AND RELEASE TIME OF FIRST TWO ZONES OF CAV 2 AND CAV 3.

CAV Index	First Zone	R_i^2 (s)	T_i^3 (s)
2	14	14.54	14.54
3	10	15.04	16.04

V. CONCLUDING REMARKS AND FUTURE

In this paper, we proposed a decentralized optimal control framework for CAVs for two interconnected intersections using scheduling theory. We formulated the control problem and provided a solution that can be implemented in real time. Upon entering the control zones, CAV solves a scheduling problem; however, if there are multiple CAVs entering the

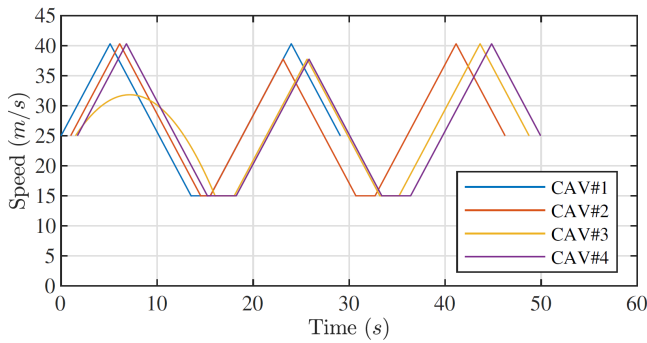


Fig. 4. Speed profile for the first 4 CAVs.

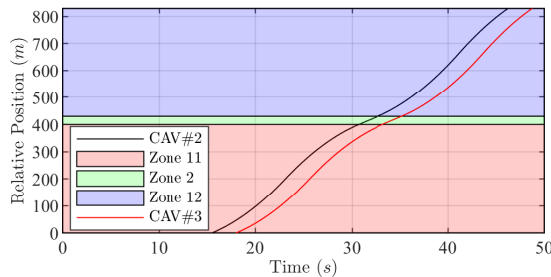


Fig. 5. Relative Position of CAV#2 and CAV#3 at zones which rear-end collision may occur.

control zone at the same time, the scheduling problem is solved sequentially with their order in the queue. The solution yields the optimal acceleration/deceleration of each CAV under the safety constraint at conflict zones. If there is no such feasible solution, then each CAV solves an energy optimal control problem. There two potential directions for the future research: (1) to involve fully constrained energy-optimal and time-optimal problems and explore when a safety constraint becomes active; and (2) to address uncertainty in data communication and control process.

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